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Hierarchical distributed scenario-based model predictive control of interconnected microgrids

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Electric power networks are changing

https://pixabay.com/en/power-station-energy-electricity-374097/

Today

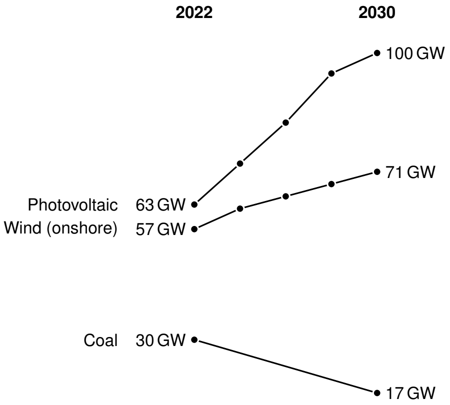
https://pixabay.com/en/solarpark-wind-park-renewable-energy-1288842/

Future

Energy systems are changing

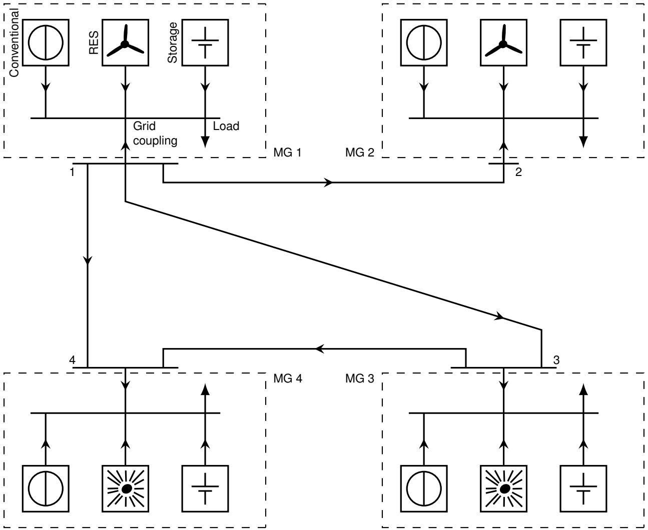
– Increasing amount of renewable generators
– Transition
  - from a small number of large-scale units
  - to a large number of small-scale units
– Uncertainty in generation will increase with installed renewable units

⇒ Need to cope with fluctuations and changing structure

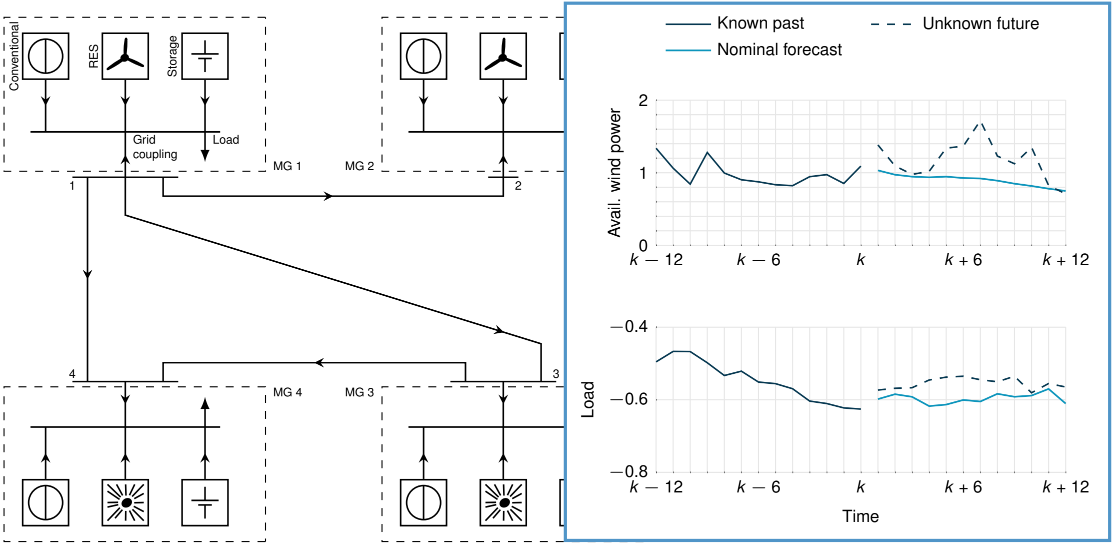


Sources: [Kohleausstiegsgesetz, 2020, Erneuerbare-Energien-Gesetz, 2021]

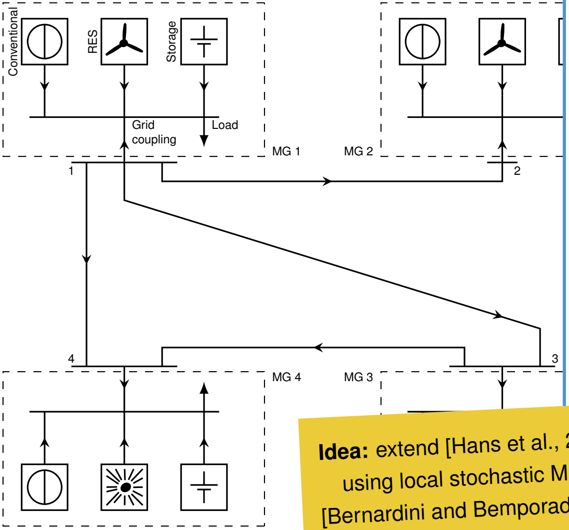
Future power networks

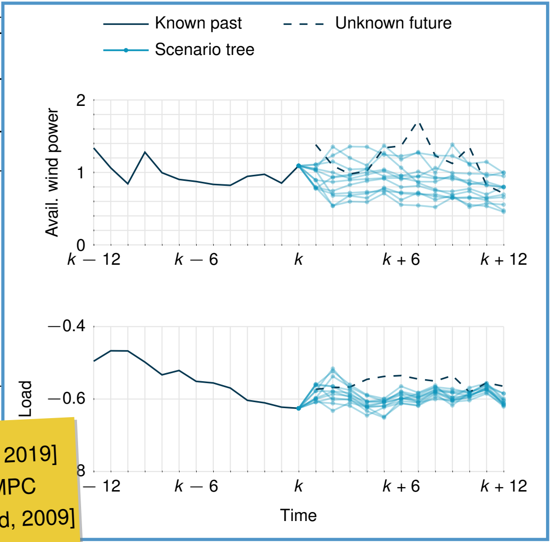


Existing certainty equivalence hierarchical distributed MPC [Hans et al., 2019]



Novel scenario-based hierarchical distributed MPC [Schenck and Hans, 2024]

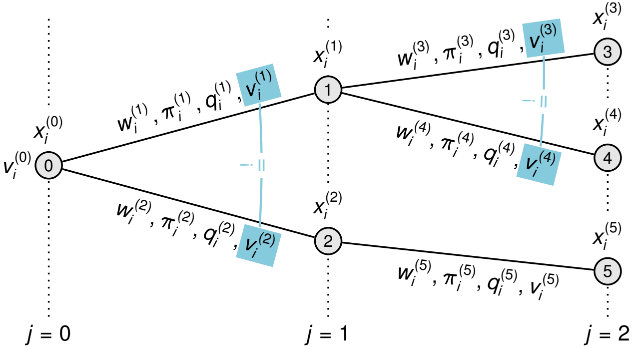




Idea: extend [Hans et al., 2019] using local stochastic MPC [Bernardini and Bemporad, 2009]

Contents

Modelling of uncertainties



– Probabilities of tree (by design)
  - Probabilities of stage $j \in \mathbb{N}_{[0,J]}$

$$\sum_{m \in \text{nodes}_i(j)} \pi^{(m)} = 1 \tag{1a}$$

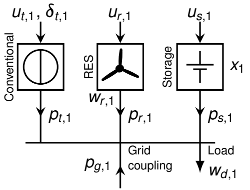
  - Probabilities of child nodes of ancestor $m \in \mathbb{M}_i \setminus \text{nodes}_i(J)$

$$\sum_{m_+ \in \text{child}_i(m)} \pi^{(m_+)} = \pi^{(m)} \tag{1b}$$

– Nonanticipativity constraint

$$v_i^{(m)} = v_i^{(n)} \quad \forall n \in \text{child}_i(\text{anc}_i(m)). \tag{2}$$

Power- and setpoint-related constraints



Decision variables of MG $i \in \mathbb{I}$

$$\text{Control input} \quad v_i = [\underbrace{u_{t,i}^T,\ u_{s,i}^T,\ u_{r,i}^T}_{\text{Power setpoints}},\ \overbrace{\delta_{t,i}^T}^{\text{On/off var.}}]^T$$

$$\text{Power} \quad p_i = [p_{t,i}^T,\ p_{s,i}^T,\ p_{r,i}^T,\ p_{g,i}]^T$$

$$\text{Stored energy} \quad x_i$$

$$\text{Uncertain input} \quad w_i = [w_{d,i}^T,\ w_{r,i}^T]^T$$

– Renewable energy sources

$$p_{r,i}^{\min} \leq u_{r,i}^{(m)} \leq p_{r,i}^{\max}, \tag{3a}$$
$$p_{r,i}^{\min} \leq p_{r,i}^{(m)} \leq p_{r,i}^{\max}, \tag{3b}$$

– Storage units

$$p_{s,i}^{\min} \leq u_{s,i}^{(m)} \leq p_{s,i}^{\max}, \tag{4a}$$
$$p_{s,i}^{\min} \leq p_{s,i}^{(m)} \leq p_{s,i}^{\max}, \tag{4b}$$

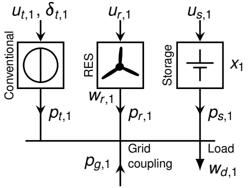
– Conventional rotating units

$$\operatorname{diag}\left(p_{t,i}^{\min}\right)\delta_{t,i}^{(m)} \leq u_{t,i}^{(m)} \leq \operatorname{diag}\left(p_{t,i}^{\max}\right)\delta_{t,i}^{(m)}, \tag{5a}$$
$$\operatorname{diag}\left(p_{t,i}^{\min}\right)\delta_{t,i}^{(m)} \leq p_{t,i}^{(m)} \leq \operatorname{diag}\left(p_{t,i}^{\max}\right)\delta_{t,i}^{(m)}. \tag{5b}$$

– Point of common coupling (PCC)

$$p_{g,i}^{\min} \leq p_{g,i}(j) \leq p_{g}^{\max}. \tag{6}$$

Modelling of dynamical system behaviour



Decision variables of MG $i \in \mathbb{I}$

Control input $\quad v_i = [\underbrace{u_{t,i}^T,\ u_{s,i}^T,\ u_{r,i}^T}_{\text{Power setpoints}},\ \overbrace{\delta_{t,i}^T}^{\text{On/off var.}}]^T$

Power $\quad p_i = [p_{t,i}^T,\ p_{s,i}^T,\ p_{r,i}^T,\ p_{g,i}]^T$

Stored energy $\quad x_i$

Uncertain input $\quad w_i = [w_{d,i}^T,\ w_{r,i}^T]^T$

– Storage dynamics and limits

$$x_i^{(m)} = x_i^{(m_-)} - T_s p_{s,i}^{(m)}, \tag{7}$$

$$x_i^{\min} - \sigma_i^{(m)} \leq x_i^{(m)} \leq x_i^{\max} + \sigma_i^{(m)} \tag{8}$$

– Steady-state approximations of lower control layers

- Power limit of renewable energy sources

$$p_{r,i}^{(m)} = \min(u_{r,i}^{(m)}, w_{r,i}^{(m)}). \tag{9}$$

- Power sharing of grid-forming storage & conventional

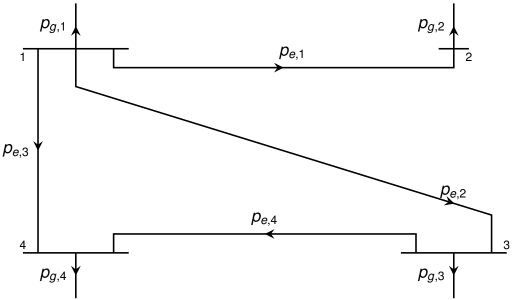
$$\operatorname{diag}(\chi_{i,1}, \dots, \chi_{i,T_i})^{-1}(p_{t,i}^{(m)} - u_{t,i}^{(m)}) = \rho_i^{(m)}\delta_{t,i}^{(m)}, \tag{10a}$$

$$\operatorname{diag}(\chi_{i,(T_i+1)}, \dots, \chi_{i,(T_i+S_i)})^{-1}(p_{s,i}^{(m)} - u_{s,i}^{(m)}) = \rho_i^{(m)}\mathbf{1}_{S_i}, \tag{10b}$$

- Power controller at the PCC

$$p_{g,i}(\underbrace{\text{stage}_i(m)}_{=j}) = -(\mathbf{1}_{R_i}^T p_{r,i}^{(m)} + \mathbf{1}_{T_i}^T p_{t,i}^{(m)} + \mathbf{1}_{S_i}^T p_{s,i}^{(m)} + \mathbf{1}_{D_i}^T w_{d,i}^{(m)}) \tag{11}$$

Modelling of interconnecting power lines



– DC power flow approximations for AC grids [Purchala et al., 2005].

$$\underbrace{\begin{bmatrix} p_{e,1}(j) \\ p_{e,2}(j) \\ p_{e,3}(j) \\ p_{e,4}(j) \end{bmatrix}}_{p_e(j)} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 1/3 & 2/3 \\ 0 & 0 & -1/3 & 1/3 \end{bmatrix}}_{\tilde{F}} \underbrace{\begin{bmatrix} p_{g,1}(j) \\ p_{g,2}(j) \\ p_{g,3}(j) \\ p_{g,4}(j) \end{bmatrix}}_{p_g(j)} \tag{12a}$$

$$0 = \mathbf{1}_I^T p_g(j) \tag{12b}$$

– Line power limit

$$p_e^{\min} \leq p_e(j) \leq p_e^{\max}. \tag{12c}$$

Operating costs

– Costs of individual MGs

$$\ell_i = \sum_{m \in \mathbb{L}_i} \pi_i^{(m)} \left(\ell_{r,i}^{(m)} + \ell_{s,i}^{(m)} + \ell_{t,i}^{(m)} + \ell_{sw,i}^{(m)} + \ell_{\rho,i}^{(m)} + \ell_{g,i}^{(m)} \right) \gamma^{\text{stage}_i(m)}. \tag{13}$$

– Cost for power transmission

$$\ell_e = \sum_{j=1}^{J} p_e^T(j) C_e p_e(j) \cdot \gamma^j. \tag{14}$$

Contents

Problem 1 (Central mixed-integer MPC)

$$\underset{\substack{\mathbf{V}_1,\dots,\mathbf{V}_I\\ \mathbf{X}_1,\dots,\mathbf{X}_I\\ \mathbf{Q}_1,\dots,\mathbf{Q}_I\\ \mathbf{P}_e,\mathbf{P}_g}}{\text{minimize}}\ \ell_e + \sum_{i\in\mathbb{I}} \ell_i$$

subject to
eqs. (2) to (12) for all $m \in \mathbb{L}_i$
as well as initial conditions $x_i^{(0)} = x_i(k)$, $\delta_{t,i}^{(0)} = \delta_{t,i}(k)$
for all $i \in \mathbb{I}$.

Does not scale well

Mean solver time for simple example grid already $>$ 2 minutes

Problem 2 (Central relaxed problem MPC)

$$\underset{\substack{\mathbf{V}_1,\ldots,\mathbf{V}_I\\ \mathbf{X}_1,\ldots,\mathbf{X}_I\\ \mathbf{Q}_1,\ldots,\mathbf{Q}_I\\ \mathbf{P}_e,\mathbf{P}_g}}{\text{minimize}}\ \boldsymbol{\ell}_e + \sum_{i\in\mathbb{I}} \boldsymbol{\ell}_i$$

subject to

eqs. (2) to (12) for all $m \in \mathbb{L}_i$

as well as initial conditions $x_i^{(0)} = x_i(k)$, $\delta_{t,i}^{(0)} = \delta_{t,i}(k)$

with $\delta_{t,i}^{(m)} \in [0,1]^{T_i}$ and $\delta_{r,i}^{(m)} \in [0,1]^{R_i}$ for all $m \in \mathbb{L}_i$

and all $i \in \mathbb{I}$.

Algorithm 1 (Hierarchical distributed MPC)

1. **Initialize:** At time k, $\forall i \in \mathbb{I}$, measure $x_i(k)$, $\delta_{t,i}(k)$ and obtain scenario tree.
2. **ADMM loop: for** $l = 0, \dots, l_{\max} \in \mathbb{N}$:
   (i) For all MGs $i \in \mathbb{I}$ (in parallel):
   - ≫ Solve Problem 3 in parallel to obtain $\mathbf{P}_{g,i}^{l+1}$.
   - ≫ Send $\mathbf{P}_{g,i}^{l+1}$ to central entity.

   (ii) Central entity:
   - ≫ Solve Problem 4 to obtain $\hat{\mathbf{P}}_{g}^{l+1}$.
   - ≫ Update Lagrange multipliers: $\boldsymbol{\Lambda}^{l+1} = \boldsymbol{\Lambda}^{l} + \kappa\left(\mathbf{P}_{g}^{l+1} - \hat{\mathbf{P}}_{g}^{l+1}\right)$.
   - ≫ Communicate $\hat{\mathbf{P}}_{g,i}^{l+1}$ and $\boldsymbol{\Lambda}_i^{l+1}$ to all MGs $i \in \mathbb{I}$.
   - ≫ Check termination criterion: **if** $\left(|\boldsymbol{\Lambda}^{l} - \boldsymbol{\Lambda}^{l+1}| < \epsilon \text{ and } |\mathbf{P}_{g,i}^{l} - \mathbf{P}_{g,i}^{l+1}| < \epsilon \text{ and } |\mathbf{P}_{g,i}^{l+1} - \hat{\mathbf{P}}_{g,i}^{l+1}| < \epsilon\right)$ or $l = l_{\max}$, **then** set $\mathbf{P}_{g,i}^{\star} = \hat{\mathbf{P}}_{g,i}^{l+1}$ and go to 3.
3. **Mixed-integer update:** For all microgrid (MG) $i \in \mathbb{I}$ (in parallel):
   - Solve Problem 5.

Problem 3 (Local ADMM problem at MG $i \in \mathbb{L}$)

$$\mathbf{P}_{g,i}^{l+1} \in \underset{\mathbf{V}_i,\mathbf{X}_i,\mathbf{Q}_i,\mathbf{P}_{g,i}}{\arg\min}\ \ell_i + \boldsymbol{\Lambda}_i^{l}\mathbf{P}_{g,i}^{T} + \kappa/2\|\mathbf{P}_{g,i} - \hat{\mathbf{P}}_{g,i}^{l}\|_2^2$$

subject to

eqs. (2) to (11) for all $m \in \mathbb{L}_i$

as well as initial conditions $x_i^{(0)} = x_i(k)$, $\delta_{t,i}^{(0)} = \delta_{t,i}(k)$

with $\delta_{t,i}^{(m)} \in [0,1]^{T_i}$, $\delta_{r,i}^{(m)} \in [0,1]^{R_i}$ for all $m \in \mathbb{L}_i$.

Algorithm 1 (Hierarchical distributed MPC)

1. **Initialize:** At time k, $\forall i \in \mathbb{I}$, measure $x_i(k)$, $\delta_{t,i}(k)$ and obtain scenario tree.
2. **ADMM loop: for** $l = 0, \dots, l_{\max} \in \mathbb{N}$:
   (i) For all MGs $i \in \mathbb{I}$ (in parallel):
   - ≫ Solve Problem 3 in parallel to obtain $\mathbf{P}_{g,i}^{l+1}$.
   - ≫ Send $\mathbf{P}_{g,i}^{l+1}$ to central entity.

   (ii) Central entity:
   - ≫ Solve Problem 4 to obtain $\hat{\mathbf{P}}_{g}^{l+1}$.
   - ≫ Update Lagrange multipliers: $\mathbf{\Lambda}^{l+1} = \mathbf{\Lambda}^{l} + \kappa\left(\mathbf{P}_{g}^{l+1} - \hat{\mathbf{P}}_{g}^{l+1}\right)$.
   - ≫ Communicate $\hat{\mathbf{P}}_{g,i}^{l+1}$ and $\mathbf{\Lambda}_i^{l+1}$ to all MGs $i \in \mathbb{I}$.
   - ≫ Check termination criterion: **if** $\left(|\mathbf{\Lambda}^{l} - \mathbf{\Lambda}^{l+1}| < \epsilon \text{ and } |\mathbf{P}_{g,i}^{l} - \mathbf{P}_{g,i}^{l+1}| < \epsilon \text{ and } |\mathbf{P}_{g,i}^{l+1} - \hat{\mathbf{P}}_{g,i}^{l+1}| < \epsilon\right)$ or $l = l_{\max}$, **then** set $\mathbf{P}_{g,i}^{\star} = \hat{\mathbf{P}}_{g,i}^{l+1}$ and go to 3.
3. **Mixed-integer update:** For all MG $i \in \mathbb{I}$ (in parallel):
   - Solve Problem 5.

Problem 4 (Central ADMM problem at coordinator)

$$\hat{\mathbf{P}}_{g}^{l+1} \in \underset{\hat{\mathbf{P}}_g, \mathbf{P}_e}{\arg\min}\ \ell_e - \sum_{i \in \mathbb{I}} \left(\mathbf{\Lambda}_i^{l} \hat{\mathbf{P}}_{g,i}^{T} + \kappa/2 \|\mathbf{P}_{g,i}^{l+1} - \hat{\mathbf{P}}_{g,i}\|_2^2\right)$$

subject to
eqs. (6) and (12) using $\hat{p}_g(j)$ instead of $p_g(j)$ for all $j \in \mathbb{N}_{[1,J]}$.

Algorithm 1 (Hierarchical distributed MPC)

1. **Initialize:** At time k, $\forall i \in \mathbb{I}$, measure $x_i(k)$, $\delta_{t,i}(k)$ and obtain scenario tree.
2. **ADMM loop: for** $l = 0, \dots, l_{\max} \in \mathbb{N}$:
   (i) For all MGs $i \in \mathbb{I}$ (in parallel):
   - ≫ Solve Problem 3 in parallel to obtain $\mathbf{P}_{g,i}^{l+1}$.
   - ≫ Send $\mathbf{P}_{g,i}^{l+1}$ to central entity.

   (ii) Central entity:
   - ≫ Solve Problem 4 to obtain $\hat{\mathbf{P}}_{g}^{l+1}$.
   - ≫ Update Lagrange multipliers:
     $\boldsymbol{\Lambda}^{l+1} = \boldsymbol{\Lambda}^{l} + \kappa\left(\mathbf{P}_g^{l+1} - \hat{\mathbf{P}}_g^{l+1}\right)$.
   - ≫ Communicate $\hat{\mathbf{P}}_{g,i}^{l+1}$ and $\boldsymbol{\Lambda}_i^{l+1}$ to all MGs $i \in \mathbb{I}$.
   - ≫ Check termination criterion:
     if $\left(|\boldsymbol{\Lambda}^{l} - \boldsymbol{\Lambda}^{l+1}| < \epsilon \text{ and } |\mathbf{P}_{g,i}^{l} - \mathbf{P}_{g,i}^{l+1}| < \epsilon \text{ and } |\mathbf{P}_{g,i}^{l+1} - \hat{\mathbf{P}}_{g,i}^{l+1}| < \epsilon\right)$ or $l = l_{\max}$,
     then set $\mathbf{P}_{g,i}^{\star} = \hat{\mathbf{P}}_{g,i}^{l+1}$ and go to 3.
3. **Mixed-integer update:** For all MG $i \in \mathbb{I}$ (in parallel):
   - Solve Problem 5.

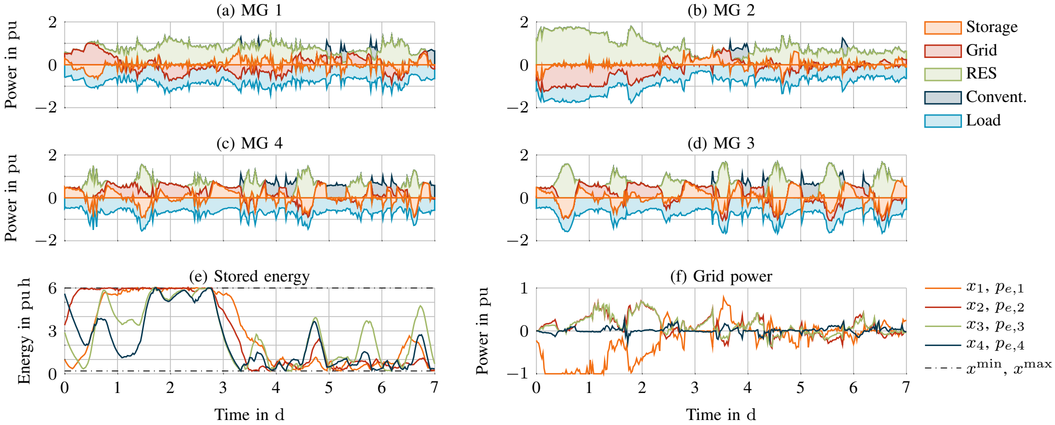
Problem 5 (Mixed-integer update at MG $i \in \mathbb{L}$)

$$\underset{\mathbf{V}_i,\mathbf{X}_i,\mathbf{Q}_i}{\text{minimize}} \ \ell_i$$

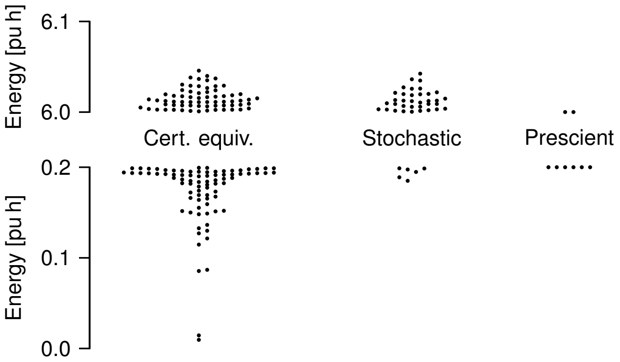
subject to
eqs. (2) to (5) and (7) to (11) for all $m \in \mathbb{L}_i$,
as well as initial conditions $x_i^{(0)} = x_i(k)$, $\delta_{t,i}^{(0)} = \delta_{t,i}(k)$
with fixed $\mathbf{P}_{g,i} = \mathbf{P}_{g,i}^{\star}$.

Contents

Closed-loop simulation results of novel approach



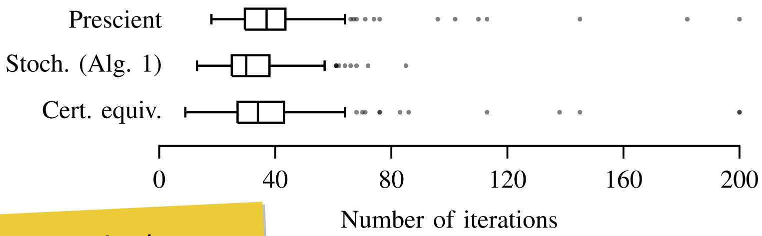
Stored energy out of desired area



Numerical comparison

		Certainty equival.	Stochastic (Alg. 1)	Prescient
Renewable energy in pu h		318.1	333.8	334.0
Conventional energy in pu h		61.0	45.2	45.9
No. of switching actions		46	40	29
Costs	MG 1	1 652.8	1 005.9	786.8
	MG 2	1 366.1	774.2	607.7
	MG 3	2 000.2	1086.1	1 003.8
	MG 4	1 755.3	1154.3	1 109.9
	Transmission	21.7	19.0	18.7
	Sum	6 796.1	4 039.5	3 527.0

Numerical properties



Algorithm 1

- Only 0.3 % higher costs compared to Problem 1
- Solve time:
  - Mean: 9 s (vs. 127 s of Problem 1)
  - Maximum: 223 s

Contents

Conclusions

- Scenario-based stochastic MPC scheme for the operation of interconnected MGs
- Distributed algorithm that reflects the hierarchical power system structure
  - local controllers are in charge of individual MGs
  - central entity is in charge of the transmission grid
- Better than certainty equivalence MPC concerning number of constraint violations and costs
- Sufficiently fast convergence

Next steps:

- Scalability
- Suboptimality
- Persistent feasibility

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