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On the Superneutrality of Money

Superneutralität des Geldes

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1. *The problem*

The determinants of the real interest rate have been a main topic in the literature on monetary theory since the days of Knut Wicksell and Irving Fisher, respectively. Is the real interest rate solely determined by non-monetary factors, such as the individual time preference rate and the population growth rate, or is it a monetary variable, which can be influenced by the monetary authority via the inflation rate? The literature discusses this problem as the "superneutrality of money". Money is said to be superneutral if – taking into account the dynamic processes of saving and capital accumulation – a change in the growth rate of the money supply leaves all steady state variables (i.e. real interest rate, real per capita income) unaffected.

The first monetary growth models (cf. Tobin, 1965, Sidrauski, 1967a) consistently denied the hypothesis of superneutrality. Tobin, for example, by employing a proportional savings function, came to the conclusion that an increase in the rate of monetary expansion increases capital accumulation and thus lowers the real interest rate (Tobin effect). His line of argument was that money and private shares of real assets are substitutes for the investor. Via higher inflation an increased money growth rate lowers the "interest" being paid on money, causing the investors to hold more real assets. As Dornbusch/Frenkel (1973) demonstrated, this result can be reversed, if increased inflation leads to a sufficiently large reduction of savings.

All these articles have been criticized, especially because of their *ad hoc* assumption on the savings behavior. It was claimed that, in general, a proportional savings function cannot be deduced from an individual utility maximization rationale. Sidrauski (1967b) was the first to employ such an optimization approach. He assumed the existence of an "immortal family", which maximizes the present value of per capita utility over all future periods. In this model, the real interest rate is invariably determined by the "Modified Golden Rule" and resistant to changes of the money growth rate.

In their overview article, Barro/Fischer (1976) hypothesize that superneutrality would not be given in case of a finite time horizon. Moreover, a second hypothesis contends that superneutrality can only occur if there are intergenerative transfers, or, to put it differently, if there is an operative bequest motive. Drazen (1981) proved the first hypothesis. Given a limited time horizon for the individual, variations in the growth of the money supply cause parallel effects for capital accumulation.

This article sets out to verify the second Barro/Fischer hypothesis. Using a monetary growth model with overlapping generations, it will be shown that the individuals counteract the Tobin effect of a higher inflation by altering the bequests to their children. A second goal is to emphasize the fact that – by changing the money growth rate – the government is in a position to decide whether money is superneutral or not.

The article is structured as follows. The next section lays out the model. Section 2.1 examines the individual optimization problem. Emphasis is placed on the microeconomic foundation of the cash-in-advance approach and on the bequest behavior. Following the description of the steady state features (Sec. 2.2), the superneutrality result will be discussed in great detail (Sec. 3). Section 4 summarizes the main results.

2. *The model*

In the literature, there are two ways of including the money functions as an accounting unit and especially as a medium of exchange into a growth theory framework: the cash-in-advance approach versus the money-in-the-utility-function approach. The first approach works by including the transaction and liquidity costs that are needed to finance consumption into an individual budget constraint. By utilizing money these costs can be lowered. In the case of the money-in-the-utility-function approach the argument is a little more subtle, or rather, less direct. The utility function of an individual is defined via the consumption and leisure time in each period of his life. In a barter economy, the search for bargaining partners, or the construction of transactional chains, requires time that could otherwise be spent on leisure activities. An economy with money reduces this time loss in favor of more leisure time. If money is inserted into the utility function, the direct utility of money reflects the increased utility caused by additional leisure time (cf. McCallum, 1983).

The vehement discussion on which approach would be more adequate (cf. Wallace, 1980, McCallum, 1983) was rendered superfluous following the publication of an article by Feenstra (1986). Both approaches are equivalent under certain preconditions. However, the proof of equivalence requires the exclusion of utility functions which are additively separable in respect to the "money" argument. In the following, the cash-in-advance approach is chosen because of its more obvious and plausible economic interpretation.

The analysis will be carried out within an overlapping generations model, in line with Barro (1974), who extended the Diamond model (1965) by adding the bequest motive.

3.1. The individual optimization problem

Each generation consists of identical individuals whose life-cycles are divided into the working and the retirement period. The utility u_t of an individual born at the beginning of period t is determined by the consumption in both periods and by the utility of its representative descendant (u_{t+1})[1]:

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[1]) As opposed to this altruistic bequest motive à la *Barro,* a strategic bequest motive can be found in the literature. In these models, in which the bequest enters the utility function (cf. *Bernheim, Shleifer, Summers,* (1985), money is never superneutral.

$$u_t = u\left(c_t^1, c_t^2\right) + \frac{1}{1+\varrho} u_{t+1} \qquad \varrho > 0 \tag{1}$$

c_t^1 and c_t^2 denote per capita consumption during the working and the retirement period; ϱ is the intergenerational discount factor. The utility function u is assumed to be strictly quasi-concave in c_t^1 and c_t^2. Since u_{t+1} includes the utility of generation t + 2 (u_{t+2}), and one argument of u_{t+2} is u_{t+3} and so on, the individual faces an infinite planning horizon despite its finite life-time.

Because the individual budget constraints are influenced by the way the government implements monetary policy, the latter shall be modelled here. It shall be assumed, that the government (central bank) pursues a policy of constant growth of the money supply:

$$M_t = (1 + \theta)\, M_{t-1}. \tag{2}$$

M_t describes the nominal money supply in period t and θ is its growth rate. The money supply θM_{t-1} to be issued at the beginning of period t is being paid as a transfer to the N_t individuals of the young generation t. Each individual receives a real transfer g_t[2]: $g_t P_t N_t = \theta M_{t-1}$. In case of a diminishing money supply, g_t will become negative and can be viewed as a lump-sum tax. P_t stands for the price level in period t. m_t^1 and m_t^2 denote the real per capita money balances in the working period (retirement period).

The first and second period budget constraints are:

$$w_t + g_t = c_t^1 + \varphi\left(c_t^1, m_t^1\right) + m_t^1 + s_t \tag{3}$$

$$\begin{aligned}(1 + r_{t+1})s_t + (1 + r_{t+1})q_{t-1} + \frac{P_t}{P_{t+1}} m_t^1 \\ = c_t^2 + \varphi\left(c_t^2, m_t^2\right) + m_t^2 + (1+n)q_t\end{aligned} \tag{4}$$

During the working period an individual supplies a fixed amount of labor, which is rewarded by the wage w_t. The individual splits the wage and the transfer into consumption and savings. Saving takes place in the form of real money balances m_t^1 and in the form of private bonds s_t (real capital). Consumption can be split into consumption of goods c_t^1 and the "transaction costs" φ needed to finance c_t^1. The resources in the retirement period consist of bond savings bearing real interest r_{t+1}, real money balances from the previous period, and the bequest q_{t-1} received from generation t − 1[3]. We assume, for simplicity's sake, that bequests only take the form of private bonds. A bequest q_{t-1} provides the individual t with $(1 + r_{t+1})q_{t-1}$ of

[2]) In accordance with the literature, it will be assumed that the individuals will regard these transfers as independent of their money balances.

[3]) The chosen modelling of bequest behavior is in its fundamentals based on *Carmichael* (1982).

retirement endowment, since private firms pay the interest rate r_{t+1} on these bonds. The resources of the retirement period are divided into consumption c_t^2, transaction costs φ, real money balances m_t^2 and the bequest $(1 + n)q_t$ an individual leaves for his $(1 + n)$ descendants. n measures the rate of growth of population:

$$N_t = (1 + n)N_{t-1} . \tag{5}$$

We shall further describe φ, the function of transaction costs. Feenstra (1986) has shown that it is feasible to integrate transaction costs into the individual budget constraint by a Baumol/Tobin as well as a Whalen type of money demand function. Feenstra's line of argument assumes an individual who strives to achieve a constant flow of consumption $\bar{c}_t$ within the period t. In order to finance consumption the individual needs money that can be obtained in exchange with bonds. The amount of bonds representing the individual's total wealth at the beginning of a period thus constantly decreases because of the need to finance consumption. Each conversion from bonds to money causes conversion costs h. If the individual minimizes the losses in interest payments caused by money balances, he will always convert the same amount from bonds into money within a period. With Z_t conversions within period t, every conversion amouns to $\mu_t = \bar{c}_t/Z_t$ and the average money balance is $\bar{m}_t = \mu_t/2 = \bar{c}_t/2Z_t$. Thus, total conversion costs are $hZ_t = h\bar{c}_t/2\bar{m}_t$. The transaction cost function φ from (3) and (4) mirrors these conversion costs. Moreover, φ can be interpreted as the expected value of "penalty costs" that would have to be paid in case of illiquidity. This case might occur if one allows for uncertainty on commodity prices. The expected value of penalty costs runs parallel to the likelihood and the amount (given proportional penalty costs) of a cash deficit. Both factors correlate positively with the planned consumption flow $\bar{c}_t$ and correlate negatively with the money balance $\bar{m}_t$.

Analogous to Feenstra, the transaction cost function φ shall possess the following properties:

$$\begin{array}{lll} \varphi \geq 0; & \varphi\,(0, m_t^i) = 0 & i = 1,2 \\ \varphi_{c^i} \geq 0 & \varphi_{m^i} \leq 0 & i = 1,2 \\ \varphi_{c^ic^i} \geq 0 & \varphi_{m^im^i} \geq 0 & i = 1,2 \end{array} \tag{6}$$

Transaction costs only occur, if consumption is positive. As outlined above, transaction costs are an increasing function of consumption and a decreasing function of the money balances. While transaction costs rise (dis)proportionally with consumption, savings in transaction costs decrease with increased money balances.

An individual maximizes (1) subject to his budget constraints (3) and (4). The individual faces four parameters of action, s_t, m_t^1, m_t^2 and q_t. Using the Lagrange method, the first-order conditions for a utility maximum are:

$$\frac{\delta u_t}{\delta c_t^1} = (1 + r_{t+1}) \frac{1 + \varphi_{c^1}}{1 + \varphi_{c^2}} \frac{\delta u_t}{\delta c_t^2} \tag{7}$$

$$1 + r_{t+1} = \frac{P_t}{P_{t+1}} \frac{1}{1 + \varphi_{m^1}} \tag{8}$$

$$\varphi_{m^2} = -1 \tag{9}$$

$$\frac{\delta u_t}{\delta q_t} \leq \frac{1+n}{1+\varphi_{c^2}} \frac{\delta u_t}{\delta c_t^2} \qquad \text{if } <, \text{ then } q_t = 0\,. \tag{10}$$

According to (7) the marginal rate of substituting c^1 for c^2 equals the transformation rate that describes the transformation process of c^1 to c^2 via money or bonds. This transformation rate includes the transaction costs. If the individual reduces one unit of consumption c^1, transaction costs are reduced as well. This reduction is countered by an increase in transaction costs if this unit is consumed during the retirement period. Only if both factors offset each other does the transformation rate correspond with the "textbook rate" $(1 + r_{t+1})$.

Equation (8) describes the portfolio selection. Savings only take the form of bonds as well as money if the rate with which resources are transferred from the working period to the retirement period is equal for both assets. The transformation rate of money comprises the "interest payment" P_t/P_{t+1} and the savings in transaction costs during the working period. The higher the inflation, the higher the savings in transaction costs need to be in order to reach equivalence with the transformation rate of bonds $(1 + r_{t+1})$.

During the retirement period, the store-of-value function of money ceases to exist for the individual. Reductions in transaction costs, however, remain as a motive for money demand. According to (9), the utility maximum is attained if an additional unit of money reduces transaction costs by also exactly one unit.

Condition (10) determines the optimal bequest. Each bequest q_t increases resources and thus the utility of the next generation. Since u_{t+1} enters u_t, the bequest also increases the utility of the bequeathing generation t. On the other hand, each bequest implies a reduction of retirement consumption in the amount of $(1 + n)q_t$ and therefore a utility loss. The optimal bequest is attained if the marginal utility gain of the bequest is equal to this marginal utility loss (adjusted to allow for the transaction costs). If the bequest motive is weak, this marginal rationale may lead to the desire to make q_t negative, i.e. to bequeath debt. If the descendant generation can refuse to accept the debt, such a negative bequest has to be excluded. Because of not sufficiently backed interest and principal payments a retired individual will not be given credit. In the model's language: Negative q_t-values are approximated by the corner solution $q_t = 0$. In this case, the sign of inequality will be effective in (10). Throughout this section we shall assume an operative bequest motive. An examination of this restriction will be given in Section 3.

For the bequeathing generation t the marginal utility gain of a bequest is defined by the increase in utility of generation t + 1 discounted by the intergenerational discount factor ϱ:

$$\frac{\delta u_t}{\delta q_t} := \frac{1}{1+\varrho} \frac{\delta u_{t+1}}{\delta q_t} \tag{11}$$

If generation t + 1 behaves as a utility maximizer, its marginal utility from the bequest amounts to (see appendix):

$$\frac{\delta u_{t+1}}{\delta q_t} = \frac{1+r_{t+2}}{1+\varphi_{c^2}} \frac{\delta u_{t+1}}{\delta c_{t+1}^2} \tag{12}$$

Substituting (12) into (11) and inserting the result into the condition for the optimal bequest (10) produces:

$$\frac{1 + r_{t+2}}{1 + \varrho} \frac{\delta u_{t+1}}{\delta c^2_{t+1}} = (1 + n) \frac{\delta u_t}{\delta c^2_t} \qquad (13)$$

The left-hand side of (13) reflects the marginal utility of bequest for generation t. As long as it is greater (less) than the opportunity costs, i.e. the utility loss from the retirement consumption, it is profitable for generation t to increase (decrease) the planned bequest.

2.2 The Steady State

To answer the question how an increased growth in the money supply will affect the real capital accumulation, a general equilibrium model is required. Solely the N_t members of the young generation are suppliers of labor. Firms produce a homogeneous commodity Y_t by means of capital K_t and labor N_t. The production function $Y_t = F(K_t, N_t)$ shall be linear-homogeneous. In the per capita version, this means

$$y_t = f(k_t) , \qquad (14)$$

with y_t describing per capita income and $k_t = K_t/N_t$ the capital intensity.

Firms maximize profits under perfect competition. The wage rate w_t and the interest rate r_t thus correspond to their marginal productivity:

$$w_t = f(k_t) - k_t f'(k_t) \qquad (15)$$

$$r_t = f'(k_t) . \qquad (16)$$

The economy examined consists of three markets: the goods market, the money market and the capital (= bonds) market. According to Walras' Law, if two of these three markets are in equilibrium, the third market must be too. In order to describe a total equilibrium of a period, one market – in our case the goods market – can be neglected.

The money supply of period t consists of three components. On the one hand, the government issues "new money" θM_{t-1} (cf. equation 2), on the other hand, the young and the old generation of the previous period transfer money to period t. The old generation transfers this money via the government. The money balances that are held at the end of the retirement period are liable to a 100 percent tax and the government returns the total sum to the economy. On the demand side, there is the young generation t with nominal money balances $m^1_t P_t N_t$ and the old generation t – 1 with nominal money balances $m^2_{t-1} P_t N_{t-1}$. The condition for money market equilibrium in period t is:

$$\begin{aligned} &\theta M_{t-1} + m^1_{t-1} P_{t-1} N_{t-1} + m^2_{t-1} P_{t-1} N_{t-2} \\ &= m^1_t P_t N_t + m^2_{t-1} P_t N_{t-1} . \end{aligned} \qquad (17)$$

Considering $M_{t-1} = m^1_{t-1}P_{t-1}N_{t-1} + m^2_{t-2}P_{t-1}N_{t-2}$ as well as $N_{t-2} = (1+n)N_{t-1} = (1+n)^2N_t$ the money market equilibrium reduces in the steady state[4]) to

$$\pi = \frac{\theta - n}{1 + n} \tag{18}$$

with the inflation rate $\pi = (P_t - P_{t-1})/P_{t-1}$. Only a money growth rate, which exceeds the population growth rate, implies a positive inflation rate.

The supply on the capital market consists of bond savings $S_t = s_tN_t$ of the young generation and of the planned bequests $(1 + n)q_{t-1}N_{t-1}$ of the old generation. The only capital demanders are firms. They have to finance the principal payments for loans from the previous period K_t and the investments $(I_t = K_{t+1} - K_t)$ in new capacities. In the steady state and in per capita terms we obtain:

$$s + q = (1 + n)k. \tag{19}$$

In order to identify properties of the steady state, we return to the transformed condition for optimal bequests (eq. 13). Since in the steady state the marginal utility of retirement consumption is equal for all generations and the interest rate is constant, (13) can be simplified to:

$$1 + r = (1 + n)\,(1 + \varrho). \tag{20}$$

Capital accumulation is already determined by the choice of optimal bequests. If the bequest motive is operative, the real interest rate is unaffected by the money growth rate θ. A change of θ will not influence capital accumulation; money is superneutral. Furthermore, since the real interest rate exceeds the population growth rate, we always get a dynamic efficient steady state. The relation $r < n$ is incompatible with an operative bequest motive (cf. Michaelis 1989).

3. *Qualifying the superneutrality result*

As assumed by Barro/Fischer, an operative bequest motive implies the superneutrality of money. The real effects of a changed money growth rate are offset by the individuals' bequest behavior. To the same extent to which the young generation reacts to increased inflation with enlarged bond savings s, the old generation diminishes planned bequests q. Since the capital intensity k is constant, equation (19) implies $ds/d\theta = -dq/d\theta$.

Neglecting, initially, the causal chain between an increased money growth rate and the amount of the transfer payment g_t, enlarged bond savings caused by inflation are explicable by two parallel effects: the substitution effect and the income effect. The substitution effect corresponds to the one from the Tobin model. Increased inflation reduces the interest rate on money, money as an asset becomes less attractive as opposed to bonds. The individuals react with a substitution in their portfolios. This substitution, however, is not performed at a 1 : 1 ratio because a m^1-decrease is accompanied by an

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[4]) In the steady state the time subscript t is omitted, since these terms are constant for all t.

increase of transaction costs as high as $\varphi_{m^1}dm^1$. Let us turn to the income effect: If the individuals did not react to increased inflation, the loss in income caused by inflation would solely reduce consumption in the retirement period. Given a concave utility function it is optimal, however, to spread the reduction of consumption over both periods. In other words, in expectation of increased inflation the individuals consume a smaller fraction of their resources $(w_t + g_t)$ during the working period and save more. With the substitution effect and the income effect working both towards more savings we shall omit its proof. It can be found in Drazen (1981).

Accelerated inflation increases via inflation tax the government's revenue, which is, however, returned as a transfer payment to the individuals:

$$g = \frac{\theta}{1+\theta}\left(m^1 + \frac{m^2}{1+n}\right).$$

The total effect of accelerated inflation on the young generation's bond savings consists of the sum of the substitution and income effects plus the effect of increased transfers. If – as assumed above – the transfer is fully paid to the young generation, the transfer effect works positively towards bond savings via an increase in income during the working period. If the transfer is fully paid to the old generation, the increased income might offset the income reduction caused by inflation during the retirement period. Whether this is the case or not unfortunately depends on the details of the chosen model: a stationary economy versus a growing one; on the question whether the young generation and/or the old generation demands money balances (see also Orphanides/Solow 1990). In the case of an increased retirement income individuals react by reducing savings during the working period. The implication is that, in contrast to the Tobin-effect, increased inflation hinders real capital accumulation. It is rather disturbing that the Tobin-effect reacts so sensitively to the distribution of the seigniorage transfers since this violates a core principle of monetary growth theory. As will be outlined below, the question of superneutrality will not be affected by various assumptions regarding the distribution of transfers.

The individuals' saving behavior as a reaction to increased money growth is not exogenous for the government but can be roughly steered by the administrative distribution of transfers. We shall further assume that transfers will be paid to a sufficient degree to the young generation. If the government increases the money growth rate θ in such a situation, capital accumulation rises and interest rates fall in accordance with (16). This, in turn, influences the bequest behavior. Since interest payments on inherited bonds will fall, growth in resources of the inheriting generation $t + 1$ will decline as well. As a consequence, the utility level u_{t+1} drops and – being an argument in the utility function of the bequeathing generation $t - u_t$ falls too. Thus, for the bequeathing generation t the marginal utility of a bequest is reduced. The generation t reacts with a reduction of planned bequests, the supply of capital (= bond savings) of the old generation is reduced. This counteracts increased capital accumulation and interest rates rise. Generation t reduces its bequests until the marginal condition (10) and equation (20) are fulfilled. Optimal bequests with respect to the utility level are achieved if the old steady state with its original capital endowment and its old interest rate has been reestablished. Reduced bequests perfectly counter the positive effect of an increased money growth rate.

If – because of a different distribution of transfers – individuals react to accelerated inflation with a reduction of bond savings, the same causal chain with opposite signs will be observable. Since the rise of the interest rate increases the marginal utility of a bequest, planned bequests are enlarged. This process comes to a hold when the marginal condition (10) and the old steady state are reestablished.

The superneutrality of money only applies for capital accumulation. In contrast to the Ricardian equivalence theorem, which claims that not only capital accumulation but also consumption in both life periods and thus the utility level is independent of the amount of government debt, a change in the money growth rate does affect the splitting of consumption between the two life periods. As opposed to the Sidrauski-model, a general statement on the development of the steady state utility is not possible. In the Sidrauski-model, consumption and real money balances are the arguments of the utility function. The utility level falls clearly since an increased money growth rate leaves consumption unaffected but real money balances decline.

The above line of argument is based on the premise of an operative bequest motive. This raises the question of the conditions under which the bequest motive will induce positive bequests. As an endogenous variable, the bequest is a function of all model parameters. The lower the individual time discount rate, or, to put it differently, the greater the influence of retirement consumption in the utility function, the higher the opportunity costs of bequests and ceteris paribus the lower the bequests. If the utility of the descendant generation $t + 1$ enters the utility function of generation t with an only small impact (high intergenerational discount factor ϱ), ceteris paribus bequests are respectively low.

The link between the government's policy parameter "money growth rate θ" and optimal bequests q is particularly interesting. If sufficient transfers flow to the young generation, the individuals "circumvent" an increase in θ by a reduction of their bequests. Since negative bequests, i.e. bequeathing debt, have to be excluded, the individuals will – given constant increases in θ – finally end up at the corner solution $q = 0$: despite a bequest motive there are no bequests. Opportunity costs in the form of reduced consumption are higher. In such a case, the individuals would like to react to an increase in θ by reducing q, but they simply cannot. Hence, θ-increases trigger the same processes as in a model without a bequest motive, they enhance capital accumulation.

This result has some interesting implications for monetary policy. If the money growth rate is below this critical value, money is superneutral; if θ is above it, money causes real effects. In other words: It is the government's decision to make money superneutral or not by simply fixing θ[5]).

The previous analysis reveals striking parallels between the superneutrality of money and the Ricardian equivalence theorem. Both neutrality results require intergenerational transfers between all future generations as a necessary condition. If this infinite chain "breaks", all postulates of neutrality must be dropped. All arguments in favor of this corner solution put forward in the debate on Ricardo's theorem (see Tobin, 1980) could be revived against the superneutrality of money. Individuals without children or

[5]) This theoretical argument multiplies the problems of testing the empirical evidence of the hypothesis of superneutrality. Very often, the starting point for these tests is the *Fisher* parity. In the case of superneutrality, changes in the inflation rate are only reflected by changes in the nominal interest rate, with the real interest rate remaining constant. If superneutrality is a function of the inflation rate, it is not surprising that different empirical results are obtained for different time samples and different countries (for an overview see *Halliassos/Tobin,* 1990, with the literature listed there).

without interest in their well-being as well as individuals with different time discount rates serve as examples. The strategic bequest motive à la Bernheim/Shleifer/Summers (see footnote 1) also implies an "imperfect" bequest mechanism. The individuals react, however, to changes in the money growth rate with changed bequests, but they do not perfectly counteract the real effects. Weil (1991) analyses further links between Ricardian debt neutrality and monetary superneutrality.

Further sources of non-superneutrality are forwarded in the literature. Superneutrality is not given if money as an argument in the production function influences the marginal productivity of capital (see Fischer, 1974). Brock (1974) expands the Sidrauski-model by a work/leisure decision. If the labor supply of the young generation is a function of the wage rate, variations of the money growth rate affect labor supply and thus capital accumulation. If the cash-in-advance constraint applies not only to consumption goods, but also to investment goods, then money and capital are complementary and a higher inflation rate implies a capital decumulation (see Stockman, 1981). Fischer (1979) focusses on the transition path to the steady state. In an approach related to the Sidrauski-model, he shows that money is superneutral in the steady state but causes real effects beyond it. Equation (13) illustrates this case. Beyond the steady state the marginal utility of the retirement consumption differs between the generations.

4. *Result*

Sidrauski's result of superneutrality has aroused new interest in the links between money, inflation and growth, since it contradicts the formerly prevalent notion that an accelerated inflation enhances capital accumulation. Barro/Fischer hypothesized that superneutrality can be reproduced in a model with overlapping generations and an operative bequest motive. As our analysis shows, this hypothesis is correct.

If the government decreases the return on money through an increased money growth rate, individuals will alter the form of their savings, substituting money for real capital. The fall in the interest rate caused by increased capital accumulation reduces the marginal utility of bequest for the bequeathing generation, thus limiting bequests and capital supply. Since this reduction, in turn, compensates the respective increase of the young generation's capital supply, total capital accumulation is invariable to changes in the money growth rate or inflation.

For monetary policy, however, the question of superneutrality in not an exogenous matter. Since individuals react to a continued increase in the money growth rate by a continuously reducing bequests, there is a critical money growth rate beyond which optimal bequests are zero or negative. If the money growth rate is below this value, money is superneutral; if it is above this value, individuals plan to circumvent capital accumulation through reduced bequests, but they simply cannot because of the assumption of no negative bequests. In other words: It is the government's decision to make money superneutral or not by choosing the adequate money growth rate.

Appendix

Generation t + 1 is able to split its extra endowment into consumption c^1_{t+1}, c^2_{t+1}, into money balances m^1_{t+1}, m^2_{t+1}, and into an increased bequest to generation t + 2. Therefore, the marginal utility from the bequest amounts to:

$$\frac{\delta u_{t+1}}{\delta q_t} = \frac{\delta u_{t+1}}{\delta c^1_{t+1}} \frac{dc^1_{t+1}}{dq_t} + \frac{\delta u_{t+1}}{\delta c^2_{t+1}} \frac{dc^2_{t+1}}{dq_t} + \frac{\delta u_{t+1}}{\delta m^1_{t+1}} \frac{dm^1_{t+1}}{dq_t} + \frac{\delta u_{t+1}}{\delta m^2_{t+1}} \frac{dm^2_{t+1}}{dq_t} + \frac{\delta u_{t+1}}{\delta q_{t+1}} \frac{dq_{t+1}}{dq_t} \tag{A1}$$

If generation t + 1 behaves as a utility maximizer, the splitting of the bequest follows the first-order conditions for a utility maximum:

$$\frac{\delta u_{t+1}}{\delta c^1_{t+1}} = (1 + r_{t+2}) \frac{1 + \varphi_{c^1}}{1 + \varphi_{c^2}} \frac{\delta u_{t+1}}{\delta c^2_{t+1}} \tag{A2}$$

$$\frac{\delta u_{t+1}}{\delta m^1_{t+1}} = \frac{1}{1 + \varphi_{c^2}} \left[(1 + r_{t+2})(1 + \varphi_{m^1}) - \frac{P_{t+1}}{P_{t+2}}\right] \frac{\delta u_{t+1}}{\delta c^2_{t+1}} \tag{A3}$$

$$\frac{\delta u_{t+1}}{\delta m^2_{t+1}} = \frac{1}{1 + \varphi_{c^2}} [1 + \varphi_{m^2}] \frac{\delta u_{t+1}}{\delta c^2_{t+1}} \tag{A4}$$

$$\frac{\delta u_{t+1}}{\delta q_{t+1}} = \frac{1 + n}{1 + \varphi_{c^2}} \frac{\delta u_{t+1}}{\delta c^2_{t+1}} \tag{A5}$$

Inserting (A2) – (A5) in (A1) produces:

$$\frac{\delta u_{t+1}}{\delta q_t} = \frac{\delta u_{t+1}}{\delta c^2_{t+1}} \left[(1 + r_{t+2}) \frac{1 + \varphi_{c^1}}{1 + \varphi_{c^2}} \frac{dc^1_{t+1}}{dq_t} + \frac{dc^2_{t+1}}{dq_t} + \frac{1 + n}{1 + \varphi_{c^2}} \frac{dq_{t+1}}{dq_t} + \frac{1}{1 + \varphi_{c^2}} \left[(1 + r_{t+2})(1 + \varphi_{m^1}) - \frac{P_{t+1}}{P_{t+2}}\right] \frac{dm^1_{t+1}}{dq_t} + \frac{1}{1 + \varphi_{c^2}} (1 + \varphi_{m^2}) \frac{dm^2_{t+1}}{dq_t}\right] \tag{A6}$$

According to its budget constraint generation t + 1 can spend the increased bequest for a rise of c^1_{t+1}, c^2_{t+1}, m^1_{t+1}, m^2_{t+1} and q_{t+1}:

$$(1 + r_{t+2})(1 + \varphi_{c^1}) \frac{dc^1_{t+1}}{dq_t} + \left[(1 + r_{t+2})(1 + \varphi_{m^1}) - \frac{P_{t+1}}{P_{t+2}}\right] \frac{dm^1_{t+1}}{dq_t} + (1 + \varphi_{m^2}) \frac{dm^2_{t+1}}{dq_t} + (1 + \varphi_{c^2}) \frac{dc^2_{t+1}}{dq_t} + (1 + n) \frac{dq_{t+1}}{dq_t} = 1 + r_{t+2} \tag{A7}$$

Regarding (A7) equation (A6) can be reduced to:

$$\frac{\delta u_{t+1}}{\delta q_t} = \frac{1 + r_{t+2}}{1 + \varphi_{c^2}} \frac{\delta u_{t+1}}{\delta c^2_{t+1}} \tag{A8}$$

Equation (A8) corresponds to equation (13) in the text.

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Summary

The paper examines the question of superneutrality of money in the context of an Overlapping Generations Model where money is introduced via a transaction technology and where agents have a bequest motive. It is shown that in the presence of an operative bequest motive an increase in the money growth rate has no impact on the accumulation of real capital. If the money growth rate exceeds a certain limit, the individuals plan to reduce bequests, but they cannot because of the assumption of no negative bequests. Therefore it is the government's decision to make money superneutral or not by choosing the adequate money growth rate.

Zusammenfassung

Im Rahmen eines monetären OG-Modells mit Vererbungsmotiv wird der Frage nachgegangen, inwieweit ein forciertes Geldmengenwachstum die Realkapitalbildung beeinflußt. Wie sich zeigt, bedarf es für die Superneutralität des Geldes der Wirksamkeit des Vererbungsmotivs. Die erhöhte Realkapitalbildung der jungen Generation (Tobin-Effekt) wird vollständig kompensiert durch einen Rückgang der geplanten Vererbung seitens der alten Generation. Überschreitet die Geldmengenwachstumsrate jedoch einen bestimmten Schwellenwert, so kommt es zur Randlösung, d.h. trotz Vererbungsmotivs treten keine Vererbungen auf. Auf die Realkapitalbildung wirkt nurmehr der expansive Tobin-Effekt. Via Festlegung der Höhe der Geldmengenwachstumsrate liegt es folglich in der Hand der Geldpolitik, darüber zu entscheiden, ob Geld superneutral ist oder nicht.

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