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CBDC as Competitor for Bank Deposits and Cryptocurrencies

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CBDC as Competitor for Bank Deposits and Cryptocurrencies

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Abstract: Private cryptocurrencies allow for payments without the need for a financial institution. These institutions, the central bank and retail banks, may thus observe a decline in the demand for their payments systems, i.e. cash and deposits. Using the monetary search model of Lagos and Wright (2005), we show that the central bank is able to tilt the playing field until it wins. By introducing an interest-bearing central bank digital currency (CBDC), the central bank is able to provide a payment system which is superior to cryptocurrencies. Miners cannot match the CBDC rate and go bankrupt. Retail banks, on the other hand, face lower profits but survive in the equilibrium. In addition, it can be welfare-improving to kick out cryptocurrencies by an interest-bearing CBDC.

JEL classification: E41, E42, E51, E52, E58

Keywords: CBDC, cryptocurrencies, welfare analysis

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1 Introduction

An increasing number of digital payments such as cryptocurrencies, corporate currencies (e.g. Diem) and mobile applications (e.g. the Swedish mobile app Swish) enhance competition on the market for payment systems. If these technological innovations better match customer preferences than traditional payment systems such as cash and bank deposits, they will be more than a short-run phenomenon. Market shares of cash and bank deposits may erode even in the long run. In this paper, we consider a world, where the central bank responds to this development by the introduction of an interest-bearing central bank digital currency (CBDC), the interest rate may serve as a new lower bound for any payment system. Does, as a consequence, cash disappear? How do retail banks adjust the interest rate for deposits to curb a deposit outflow toward CBDC and cryptocurrencies, how is the interest rate for loans affected? On the other hand, is the central bank able to destroy the business model of cryptocurrency miners, since miners are forced to lower transaction fees, can the central bank enforce the bankruptcy of miners? This paper sheds light on these questions.

From a customer point of view, the choice of a payment system is no either or decision. Typically, customers use two or more payment systems simultaneously. Decisive features of a payment system are the acceptance rate, storage costs, the real rate of return, the degree of anonymity, transactions fees, payment speed and security (see, e.g., Bagnall et al., 2016; and Mancini-Griffoli et al., 2019). The comparative disadvantage of cash is a negative real rate of return and high transaction costs (for large-value and long-distance payments), the comparative disadvantage of deposits is the low degree of anonymity and the low speed of (cross-border) transfers. Digital payment systems aim to eliminate these weaknesses. However, both central banks as provider of cash and retail banks as provider of deposits did not sit back and wait. Central banks are exploring the pros and cons of a CBDC (see, e.g., Sveriges Riksbank, 2021), retail banks are improving the payment structure which speed up and simplifies transactions (see Bech and Hancock, 2020; and Blocher et al., 2017).

Analyzing (digital) payment systems has become a cottage industry in monetary economics, for cryptocurrencies see Böhme et al. (2015) and John et al. (2022), for a CBDC see Meaning et al. (2021), for corporate currencies see Zetzsche et al. (2021) and Hanl (2022), for retail and wholesale payment systems see Petralia et al. (2019). The interaction between different payment systems, however, is less well explored. The exceptions are restricted mostly to the analysis of just two payment systems. Bindseil et al. (2021) focus on the interaction between an interest-bearing CBDC and bank deposits. To reduce the probability of a bank run, the central bank should set an upper bound for CBDC deposits. Chiu et al. (2021) show that the profit-maximizing response of a retail bank is an increase in the interest rate for bank deposits. Fernández-Villaverde and Sanches (2019) focus on the competition between privately issued fiat monies. Schilling and Uhlig (2019) model the competition between traditional cash and a privately-issued cryptocurrency.

To get a more comprehensive overview of the interactions, this paper models cash, a CBDC, bank deposits and cryptocurrencies simultaneously. The investigation of repercussions and feedback effects requires an environment with a central bank (providing cash and CBDC), retail banks (providing deposits and loans) and miners (providing cryptocurrencies). To this end, we augment the search-theoretic model of Lagos and Wright (2005).

In this paper, we show that the central bank is able to tilt the playing field until it wins. If the central bank provides an interest-bearing CBDC, retail banks and miners are forced to match the CBDC rate to avoid runs. Retail banks will do this by a mixture of higher deposit rates, higher loans rates and a decline in profits. Miners, however, go bankrupt since they are not able to offer such conditions. We show that such a kickoff of miners may be welfare-improving.

This article is organized into five parts. Sections 2 reviews the literature, in particular, the literature dealing with digital payment systems as part of monetary search models. Section 3 describes our framework. Section 4 analyses how a CBDC affects the business of retail banks and miners. A welfare analysis is made in Section 5. Section 6 concludes.

2 Literature Review

As Fernández-Villaverde (2018) already argued, to talk about money means to talk about trading frictions; the former exists because of the latter. Compared to cash-in-advance or money-in-the-utility-function models, money has an explicit role in monetary search models: namely, simplify trade by minimizing trade frictions. Some of the monetary search models, in particular, models with digital money, are briefly mentioned in the following.

Lagos and Wright (2005, henceforth LW) set the stage for monetary search models. LW focus on the effects of discounting and bargaining power on consumption. Both issues cause inefficiencies so that the first best allocation is not reached. If tomorrow's consumption needs present-day money and money is discounted, private agents will choose less money than necessary to buy the welfare-maximizing quantity of goods. Similarly, if the seller of a good has some power in the bargain over the price of the good, the price will exceed marginal costs. Thus, in general, the first best allocation is reached only if there is no discounting and buyers have complete bargaining power.

Fernández-Villaverde and Sanches (2019) implement cryptocurrencies in LW to investigate the competition between privately-issued cryptocurrencies. If the marginal costs of issuing a new currency are zero, there will be no competitive equilibrium. Extending the model by government money ensures an equilibrium, but any equilibrium with private money is inefficient. In this way, the portfolio of private and government monies has a positive (negative) return if the overall money supply is shrinking (growing). As long as private agents value private cryptocurrencies, a government fails to implement the Friedman rule since private miners do not retract their previously issued coins. Instead, they will issue further coins so that the government is unable to reach its overall money supply target.

Chiu et al. (2021) use the LW framework to study the effects of an interest-bearing CBDC on retail banks. If retail banks have no market power, issuing an interest-bearing CBDC would crowd out retail banks. If, on the other hand, retail banks have some market power, issuing an interest-bearing CBDC forces retail banks to increase their deposit rate to keep their customers. In this case, retail banks are able to finance the higher interest rate due to their profits. As a consequence, monopoly profits and thus inefficiencies decrease so that consumption and welfare increase.

Davoodalhosseini (2021) also implements a CBDC in LW to distinguish between three scenarios: only cash, only a CBDC or both systems are available. The main advantage of cash is that it is anonymous. The main advantage of a CBDC is that it is interest-bearing. As long as the (anonymous) costs for using a CBDC do not exceed a well-defined threshold, the "CBDC only" scenario is welfare-maximizing. If both cash and CBDC are available, welfare may decline.

Almosova (2018a) uses LW to show that cryptocurrencies are able to set an upper limit for inflation. If costs for emission of cryptocurrencies are manageable, prices in transactions with cryptocurrencies are also limited. Thus, there is an upper limit for the money growth rate and thus inflation. Moreover, Almosova (2018b) uses LW to demonstrate that market tightness, defined as ratio between demanded transactions and miners, affects trade probability negatively. In this case, money demand is hump shaped: if the return for (providing) money is sufficiently high, there are enough miners and precautionary money demand decreases.

There is also some non-search-theoretic literature on a CBDC. Barrdear and Kumhof (2021) use a DSGE approach to calculate the impact on welfare if an interest-bearing CBDC is issued for government bonds. The issued volume of a CBDC is 30 percent of GDP. In this case, a CBDC is able to raise GDP by up to three percent. The reason is a more efficient process of financial intermediation. Bordo (2021) points to some more benefits, firstly, the effective lower bound can be eliminated, secondly, price stability can be reached more easily, and thirdly, a CBDC can facilitate international transactions.

The ECB (2020) emphasizes that a CBDC could be necessary to secure demand for legal payment systems, at least if demand for cash declines. In this way, a CBDC complements cash, it does not substitute it. Nevertheless, there is no incentive to crowd out private solutions for efficient digital retail payments. Meaning et al. (2021) investigate the impact of a CBDC as an additional monetary policy instrument. As long as a CBDC is interest-bearing, the central bank is able to act as usual by determining the interest rate and the money supply of a CBDC.

Alongside these, there is some literature on the impact on retail banks. Agur et al. (2022) distinguish between cash, deposits and a CBDC. If a CBDC is closely related to cash, there is the danger that cash will disappear. If, on the other hand, a CBDC is similar to deposits, maturity transformation of commercial banks is at risk. Thus, there is a trade-off for the central bank: either cash or commercial banks are endangered. Andolfatto (2021) assesses the impact of an interest-bearing CBDC on a monopolistic retail bank sector and emphasizes that the introduction of a CBDC increases competition. Since the retail bank sector has to offer a higher deposit rate to keep its deposits, profits decrease.

Bindseil (2020) attests that the benefits of a CBDC include a more efficient retail payment system and a stronger monetary policy. Risks include, in particular, retail bank runs in crisis situations. In this case, a two-tier remuneration of a CBDC minimizes that risk. Chiu and Davoodalhosseini (2021) distinguish between two CBDC types: a cash-like type (non-interest-bearing) and a deposit-like type (interest-bearing). Depending on the type, the effects on welfare and bank intermediation differ. A cash-like CBDC is more able to promote consumption and thus welfare. Additionally, even in the absence of bank market power, a cash-like CBDC is able to increase bank intermediation by 5.8%. On the other hand, a deposit-like CBDC promotes bank disintermediation by 2.6%.

Fernández-Villaverde et al. (2021) show with a Diamond and Dybvig model that the central bank can become the deposit monopolist by providing a CBDC. This endangers maturity transformation of retail banks. Kumhof and Noone (2018) show that risks for retail bank runs are manageable, as long as some core principles for a CBDC are fulfilled. The core principles include an adjustable interest rate for a CBDC and a limited acquisition. For instance, a CBDC can only be acquired in exchange for government bonds at the central bank. Williamson (2021) studies the effects of a CBDC, which replaces cash, on financial stability and economic welfare. As long as transactions with a CBDC are more convenient, the probability of bank runs increases. Nevertheless, economic welfare can be higher since the gain from a CBDC exceeds the loss that occurs due to financial instability.

3 Framework

3.1 Environment

The framework mainly builds on Lagos and Wright (2005) while Fernández-Villaverde and Sanches (2019) and Chiu et al. (2021), respectively, provide the cryptocurrency and banking background, respectively. Each period is divided into two sub-periods, day and night. The discount factor between two periods is $\beta \in (0, 1)$. In the day, there is a decentralized bilateral matching market (DM) where only private agents act. At night, there is a centralized market (NM) where retail banks, entrepreneurs, miners and the central bank also act. On DM *special* goods are consumed where price and quantity are determined in a Nash bargaining process. Agents of type $j \in \{1, ..., J\}$, with $J \geq 3$, prefer special goods of type j but produce goods of type $j + 1$ (modulo J). Thus, nobody consumes its own production and pure barter does not take place, see also Matsuyama et al. (1993). This implies that money is necessary for trade on DM. In contrast, on NM a *general* good is consumed by everyone, the price of the general good is one. Before considering each group in detail, the overall environment is explained in brief.

The environment consists of private agents, retail banks, entrepreneurs, miners and the central bank, who are all connected through different transactions, see figure 1.

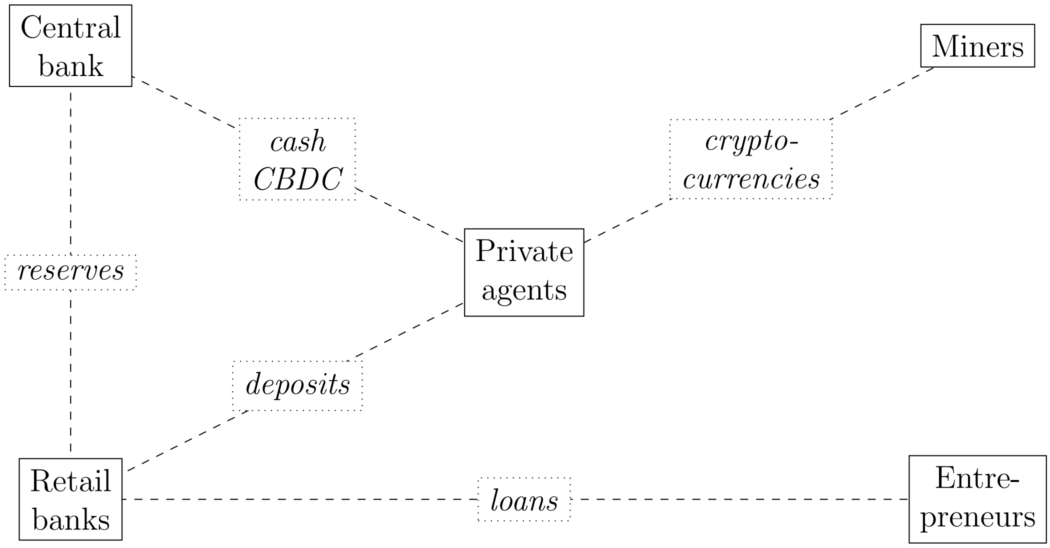

Figure 1: The Environment

First, private agents use cash ($i = 1$), CBDC ($i = 2$), deposits ($i = 3$) and cryptocurrencies ($i = 4$) to purchase special goods on DM[1]. Afterwards, the money that is left over from DM is used to finance the general good on NM and the money portfolio for the next period. Second, retail banks use deposits from private agents for the loan business and the minimum reserve at the central bank. Only retail banks are able to offer loans to entrepreneurs since they are the only group that is able to reclaim the money. The profits from the loan business are used for the general good. Third, entrepreneurs use loans from retail banks as investment capital. Compared to the residual groups, entrepreneurs have the knowledge to multiply resources through investing. The gains are used to finance the general good.

[1]Each agent holds an identical currency portfolio since the cumulative distribution function (CDF) of money degenerates at the latest after the first period, see also chapter 3.2.

Fourth, miners provide cryptocurrencies as payment system for private agents. Analogous to retail banks and entrepreneurs, the profits from mining are used to finance the general good. Finally, the central bank offers cash and a CBDC as payment systems for private agents and charges a minimum reserve from retail banks. The main goal of the central bank is to secure demand for at least one legal payment system, i.e. cash or CBDC.

3.2 Private Agents

As in Lagos and Wright (2005), there is a [0,1]-continuum of private agents acting either as a buyer or seller. The Bellman equation for DM is

$$\begin{aligned} D(m) = \sum_{i=1}^{4} \underbrace{\alpha_i [N(m_i - p_i - \eta_i q_i, m_{-i}) + u(q_i)]}_{\text{purchase on DM with } i} \\ + \sum_{i=1}^{4} \underbrace{\alpha_i [N(m_i + p_i, m_{-i}) - c(q_i)]}_{\text{sale on DM for } i} + \underbrace{(1 - 2\sum_{i=1}^{4} \alpha_i) N(m)}_{\text{no trade on DM}}, \end{aligned} \tag{1}$$

where $m = (m_1, m_2, m_3, m_4)$ is the identical currency portfolio in the initial stage, with $m_i \equiv \phi_i m_i^n$ as real balances of payment system i, meaning the purchasing power per unit of i, ϕ_i, times the number of nominal units, m_i^n.

The first part of eq. (1) describes a purchase on DM, where $\alpha_i \equiv \tilde{\alpha}_i / J$ is the probability that payment system i is used by agents of type j. Here, $\tilde{\alpha}_i$ is the market share of i, with $\sum_{i=1}^{4} \tilde{\alpha}_i = 1$. Buyers have utility $u(q_i) = \frac{q_i^{1-\sigma}}{1-\sigma}$ of consuming q_i, with $\sigma \in (0, 1)$. They pay $p_i + \eta_i q_i$ and enter NM with $m = (m_i - p_i - \eta_i q_i, m_{-i})$, where $p_i \equiv \phi_i p_i^n$ is the real price, $\eta_i \equiv \phi_i \eta_i^n$ the real transaction fee and m_{-i} the real balances of the residual payment systems.

The transaction fee $\eta_i \in [0, 1)$ can be a real transaction fee for the confirmation of a transaction as well as an anonymous cost if the payment system is not fully anonymous. It is assumed that the transaction costs increase in the transferred amount. Otherwise, the transaction fee does not affect money demand. This is an extension compared to the existing literature to model the costs and the degree of anonymity of a payment system. For small transactions, there is no fee or anonymity cost for cash. On the other hand, the central bank, retail banks and miners can charge a processing fee for a transaction. Apart from cryptocurrencies, there are also anonymity costs if agents pay with a CBDC or deposits.

Things are similar for the second summand of eq. (1) which covers a sale on DM. Sellers have costs $c(q_i) = q_i$ for producing quantity q_i. They receive p_i and enter NM with $m = (m_i + p_i, m_{-i})$. Finally, the third part of eq. (1) describes the case where agents do not trade on DM so that they enter NM with the currency portfolio from the initial stage. Thus, for the NM the Bellman equation is

$$N(m) = x_P + \beta D(m^+), \quad \text{with} \quad \underbrace{\sum_{i=1}^{4} m_i}_{\text{assets}} = \underbrace{x_P + \sum_{i=1}^{4} \psi_i m_i^+}_{\text{liabilities}}. \tag{2}$$

On NM agents consume and produce a general good, where x_P is the net consumption of private agents, the difference between one unit of the general good and one unit of work with a wage of one. Thus, on NM the utility and cost function are both linear[2] with a slope of one. Afterwards, they enter DM next period with m^+[3]. As long as $x_P = 0$, agents consume as much as they work on NM so that they transfer their complete current real balances $\sum_{i=1}^{4} m_i = \sum_{i=1}^{4} \psi_i m_i^+$ into the next period.

[2]The quasi-linearity of the cost function is crucial. Otherwise, money demand for the next period is affected by the current money holdings so that the CDF of money does not degenerate after the first period.

[3]All variables with a superscripted plus embody the *next* time period.

Here, $\psi_i = \phi_i/\phi_i^+ \geq \beta$ is the price of payment system i, while $1/\psi_i$ is the return. Since ϕ_i is the purchasing power per unit, $1/\phi_i$ is equal to the price level. This implies that ψ_i is equal to the inflation rate in a trade with payment system i.

One can prove that $N(m_i, m_{-i})$ is linear in m_i by implementing the budget constraint in the Bellman equation for NM. Using this and combining eq. (1) and (2) yields

$$D(m) = \sum_{i=1}^{4} \alpha_i \underbrace{[u(q_i) - p_i - \eta_i q_i]}_{\substack{\text{trade surplus} \\ \text{for a buyer}}} + \sum_{i=1}^{4} \alpha_i \underbrace{[p_i - c(q_i)]}_{\substack{\text{trade surplus} \\ \text{for a seller}}} + x_P + \beta D(m^+). \tag{3}$$

Agents' benefit consists of a surplus from a purchase on DM (utility of consuming minus price and transaction fee), a surplus from a sale on DM (price minus costs for production), net consumption on NM and the discounted utility from m^+.

To determine q_i, the Nash bargaining product, defined by the product of a buyer's and seller's surplus of trading, is considered:

$$\max_{q_i, p_i} [u(q_i) - p_i - \eta_i q_i]^\theta [p_i - c(q_i)]^{1-\theta}.$$

To keep things tractable, it is assumed that buyers have the complete bargaining power, $\theta = 1$, and make a take it or leave it offer[4]. Thus, buyers choose the quantity where sellers are indifferent between selling or not and offer a price which covers costs so that $p_i = c(q_i)$. In this way, they maximize their trade surplus $\Delta_i(q_i) \equiv u(q_i) - p_i - \eta_i q_i > 0$. But the welfare-maximizing quantity q_i^*, which is defined by $u'(q_i^*) = c'(q_i^*) + \eta_i$, is only traded if the buyers' money holdings are sufficiently large, $m_i \geq c(q_i^*)$. Otherwise, sellers are not willing to produce q_i^* because they are not fully compensated for their costs.

[4]This is analogous to Fernández-Villaverde and Sanches (2019) or Chiu et al. (2021) and excludes inefficiencies due to the bargaining power. Thus, inefficiencies only arise from discounting.

To obtain information about money holdings the FOC of eq. (3) regarding to m_i^+ is considered. In this way, $q_i^+ = p_i^+ = m_i^+$ holds. The first part, $q_i^+ = p_i^+$, due to the take it or leave it offer. Moreover, since money gets discounted, agents do not choose more money than necessary to pay the price p_i^+ so that $p_i^+ = m_i^+$ holds. The inverse money demand function is

$$\Omega_i(m_i^+) = \underbrace{\alpha_i \left[\frac{1}{{m_i^+}^{\sigma}} - (1+\eta_i) \right]}_{\substack{\text{equal to} \\ u'(q_i^+) - [c'(q_i^+) + \eta_i]}} \quad \Leftrightarrow \quad m_i^+ = \left[\frac{1}{\frac{\Omega_i}{\alpha_i} + (1+\eta_i)} \right]^{\frac{1}{\sigma}}, \tag{4}$$

where $\Omega_i \equiv \psi_i/\beta - 1 \geq 0$ are the storage costs of payment system i. Thus, $\Omega_i = 0$ implies $u'(q_i^+) = c'(q_i^+) + \eta_i$ so that the welfare-maximizing quantity, ${q_i^+}^* = \sqrt[\sigma]{\frac{1}{1+\eta_i}}$, is traded. On the other hand, if $\Omega_i > 0$, there are storage costs for transferring money into the next period and agents are not willing to transfer the welfare-maximizing amount of money, which is necessary to buy ${q_i^+}^*$. In this case, the return $1/\psi_i$ of payment system i is too low[5].

In general, money demand increases with the trade probability and decreases with the storage costs and transaction fee, see also figure 2. An increase in the trade probability causes a rotation to the right. As a consequence, money demand is less elastic. An increase in the transaction fee causes a shift to the left. Moreover, for a given $\tilde{\Omega}_i$, money demand is the highest if the trade probability is high, $\alpha_i \to 1/3$, and the transaction fee low, $\eta_i = 0$, see the black line. In addition, there are different parameter combinations where a specific amount of money $\tilde{m}_i^+$ is demanded. For instance, if the storage costs and trade probability are high (low) and the transaction fee low (high), see the black (blue dashed) line.

[5]This also reveals two differences to the previous generation of Trejos and Wright (1995): first, the discount problem occurs on the buyer's side. And second, inefficiencies still evoke even if buyers have the complete bargaining power.

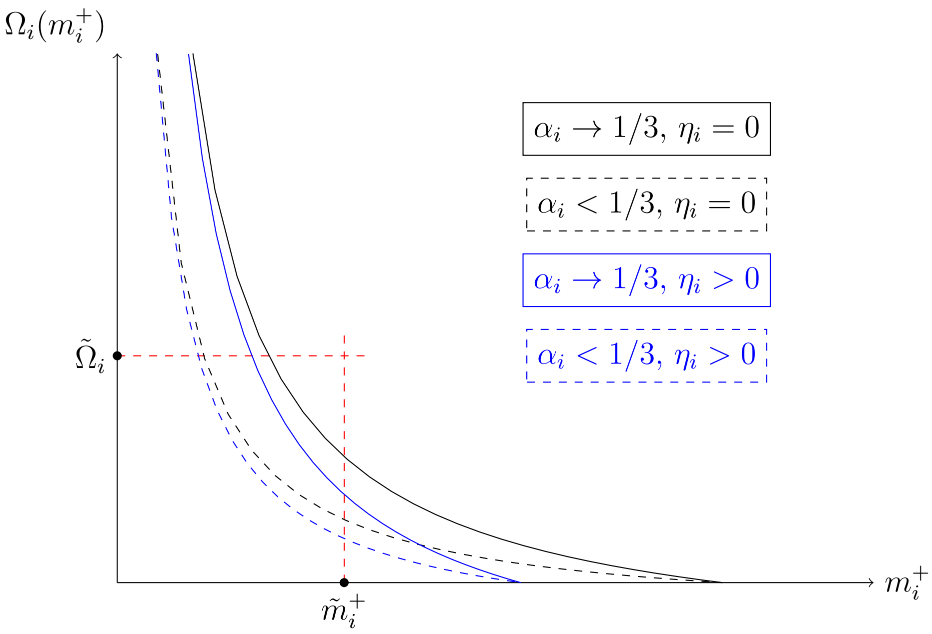

Figure 2: Money demand

Interestingly, the trade probability (transaction fee) is decisive for money demand if the storage costs are high (low). Thus, money demand functions with the same trade probability (transaction fee) converge if the storage costs increase (decrease). For instance, the black and blue line converge if the storage costs increase since both money demand functions have the same trade probability. If the storage costs are high, private agents are primarily concerned about the trade probability since a low trade probability implies a long holding duration and thus a high loss in purchasing power. On the other hand, if the storage costs are low, private agents only face a small loss in purchasing power so that they are primarily concerned about the transaction fee.

Proposition 1: *(i) The welfare-maximizing quantity q_i^* will be traded, if and only if there are no storage costs, $\psi_i = \beta$. (ii) For $\psi_i > \beta$, agents' money demand is not sufficient to buy q_i^*. In this case, the traded quantity is lower, $q_i < q_i^*$.*

Eq. (4) reveals that the CDF of money degenerates at the latest after the first period. Agents have different money holdings after DM, but they all face the same storage costs, trade probabilities and transaction fees. Due to the assumption of a quasi-linear cost function on NM, each agent chooses the same m^+. Each m_i^+ is unique since $\Omega_i'(m_i^+) < 0$ for all $m_i^+ > 0$.

Because of $u'(0) = \infty$, the expected value of money is always positive. This implies that at least one payment will be used. Rearranging the expected utility and cost

$$\underbrace{\sum_{t=1}^{\infty} \overbrace{\left(\frac{1-\alpha_i}{1+\Omega_i}\right)^{t-1}}^{\substack{\text{prob. for no} \\ \text{trade until } t-1}} \overbrace{\left(\frac{\alpha_i}{1+\Omega_i}\right)}^{\substack{\text{prob. for a} \\ \text{trade in } t}} \overbrace{\left(\frac{m_i^{1-\sigma}}{1-\sigma} - \eta_i m_i\right)}^{\text{net utility}}}_{\text{discounted exp. net utility from consumption in period } t} > \underbrace{m_i}_{\substack{\text{exp. costs} \\ \text{(for sure)}}}$$

yields $\frac{m_i \Omega'(m_i)}{1-\sigma} < 0$, which, due to the assumption $\sigma \in (0,1)$, always holds. However, private agents only use the most economical payment system(s). Using eq. (4) again, one can show that agents only use payment system i, $m_i > m_{-i} = 0$, if the overall cost difference

$$\Lambda_i^{-i} \equiv \left(\frac{\Omega_{-i}}{\alpha_{-i}} + \eta_{-i}\right) - \left(\frac{\Omega_i}{\alpha_i} + \eta_i\right)$$

between $-i$ and i is positive, $\Lambda_i^{-i} > 0$[6]. Even if the residual payment systems $-i$ have a positive expected value, they are not used in equilibrium since the expected value is below the expected value of payment system i. Due to the transaction fee, i and $-i$ do not need the same storage costs to be used simultaneously in equilibrium. This is an extension compared to the previous literature[7] where payment systems must always have the same storage costs to be used simultaneously.

[6]For $\Lambda_i^{-i} = 0$, all payment systems are used simultaneously in equilibrium, $m_i = m_{-i} > 0$.
[7]For instance, Fernández-Villaverde and Sanches (2019) or Chiu et al. (2021).

It is conceivable that payment system i has higher weighted storage costs but a lower transaction fee compared to the residual payment systems $-i$. For instance, cash and a CBDC are used simultaneously, $m_1 = m_2 > 0$, if and only if

$$\Lambda_2^1 = \underbrace{\left(\frac{\Omega_1}{\alpha_1} - \frac{\Omega_2}{\alpha_2}\right)}_{\substack{\text{weighted stor. cost} \\ \text{excess for cash}}} - \underbrace{(\eta_2 - \eta_1)}_{\substack{\text{transaction fee} \\ \text{excess for a CBDC}}} = 0.$$

In this case, the weighted storage costs for cash are higher since cash is not interest-bearing. On the other hand, the transaction fee for a CBDC is higher since a CBDC is not fully anonymous[8].

Proposition 2: *Before entering DM, money holdings are homogeneous across all agents. (i) Because of $u'(0) = \infty$, agents choose at least one payment system in equilibrium. (ii) For $\Lambda_i^{-i} > 0$, agents use only payment system i. On the other hand, for $\Lambda_i^{-i} = 0$, agents use all four payment systems simultaneously.*

3.3 Retail Banks

Next to private agents, there is a finite number of retail banks using their loan business to ensure net consumption x_B on NM, see also Chiu et al. (2021). Since only retail banks are able to reclaim loan payments, they are the only group that is able to offer loans to entrepreneurs. Retail banks use deposits $\psi_3 m_3$ from private agents for two assets: a share of $\chi \in (0, 1)$ must be used as a reserve which is deposited at the central bank. The interest rate for reserves is equal to the interest rate of cash $1/\psi_1$. Thus, one can also argue that retail banks have to hold a cash reserve. The residual share $1 - \chi$ is used for loans given to entrepreneurs where the loan rate is $\rho > 1$. Hence, the weighted return is $\Gamma \equiv \chi/\psi_1 + (1 - \chi)\rho > 0$.

[8] As already mentioned, the transaction fee η_i covers real transaction costs for a confirmation of a transaction as well as anonymity costs.

Profits $\Gamma\psi_3 m_3$ and revenues from transaction fees $\alpha_3\eta_3 m_3$ are used to finance net consumption x_B on NM and, at the end of the period, pay back deposits m_3. Since net consumption x_B is equal to the profit, retail banks maximize their profits subject to their budget constraint

$$\underbrace{\Gamma\psi_3 m_3 + \alpha_3\eta_3 m_3}_{\text{assets}} = \underbrace{x_B + m_3}_{\text{liabilities}}. \tag{5}$$

As in Chiu et al. (2021) and due to empirical evidence of Dreschler et al. (2017), it is assumed that the deposit market is non-competitive while the loan market is competitive. Thus, retail banks maximize their profit by choosing their deposits m_3. With respect to eq. (4), the FOC is

$$\psi_3(m_3) + m_3\psi_3'(m_3) = \frac{1-\alpha_3\eta_3}{\Gamma}. \tag{6}$$

Since $\psi_3(m_3) + m_3\psi_3'(m_3)$ is decreasing in m_3, retail banks increase their deposits if the trade probability, transaction fee or weighted return increases.

Next to the deposit channel there is also the loan channel. Loan supply ℓ_s depends on the interest rate for reserves and loans. Three cases are possible:

- If $\rho < 1/\psi_1$, loan supply is zero. In this case, there is no incentive for retail banks to invest in loans since the interest rate for reserves is higher.
- If $\rho = 1/\psi_1$, loan supply is between zero and $(1-\chi)\psi_3 m_3$. Since the interest rate for reserves and loans are equal, retail banks are indifferent about investing in reserves or loans. Thus, if $\ell_s < (1-\chi)\psi_3 m_3$, retail banks also hold an excess reserve.
- If $\rho > 1/\psi_1$, loan supply is $(1-\chi)\psi_3 m_3$. The loan rate exceeds the interest rate for reserves and retail banks invest all their remaining deposits in loans.

It should be mentioned that loans do not necessarily increase in ρ. If ρ increases, deposits m_3 do too, see eq. (6). But ψ_3 is decreasing in m_3, see eq. (4). Thus, it is not guaranteed that $m_3\psi_3(m_3)$ is increasing in m_3. Observing the money demand function (4), it is straightforward to show that the condition

$$\eta_3 < \frac{1-\alpha_3}{\alpha_3} + \frac{1-\sigma}{m_3^\sigma} \tag{7}$$

secures that $m_3\psi_3(m_3)$ is increasing in m_3 so that ℓ_s is increasing in ρ. Here, $\eta_3 \in [0,1)$ is sufficient for the validity of condition (7) since $\frac{1-\alpha_3}{\alpha_3} \geq 2$ and $\frac{1-\sigma}{m_3^\sigma} > 0$. Condition (7) extends Chiu et al. (2021) who focus on the case without a transaction fee, $\eta_3 = 0$. If the transaction fee is large, money demand is low and less elastic, meaning ψ_3 (respectively Ω_3) decreases significantly for a small increase in m_3. In this area $m_3\psi_3(m_3)$ is decreasing in m_3 and loan supply is decreasing in the loan rate.

Proposition 3: *(i) If the interest rate for loans exceeds the interest rate for reserves, retail banks offer loans to entrepreneurs. (ii) The loan supply increases in the loan rate if the transaction fee η_3 does not exceed the threshold given by* (7).

3.4 Entrepreneurs

The customers for the loans are a continuum of entrepreneurs. We follow Chiu et al. (2021) by assuming that entrepreneurs have an investment opportunity to transform a unit of ℓ into $f(\ell)$ units of ℓ. In order to finance the investments, entrepreneurs demand loans. The transformation is given by $f(\ell) = \frac{\ell^{1-\varepsilon}}{1-\varepsilon}$, with $\varepsilon \in (0,1)$ as investment efficiency factor. Entrepreneurs maximize their net consumption x_E subject to their budget constraint

$$x_E = f(\ell) - \rho\ell. \tag{8}$$

The solution to maximize net consumption or profit, respectively, delivers the loan demand curve

$$\ell_d = (1/\rho)^{1/\varepsilon}.$$

Demand is decreasing in the loan rate ρ and increasing in the investment efficiency factor ε. If the investment efficiency factor increases, demand shifts to the right, see the black dashed line in figure 3.

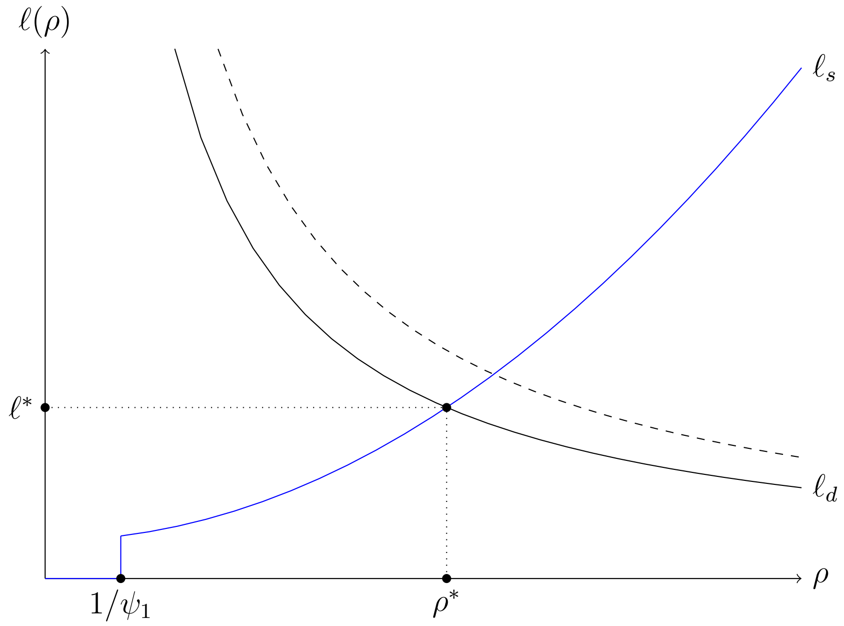

Figure 3: The loan market

The blue curve captures loan supply ℓ_s. As mentioned above, loan supply is zero as long as the interest rate for reserves exceeds the loan rate. If the rates are equal, retail banks are indifferent and loan supply is between zero and $(1-\chi)\psi_3 m_3$. Finally, if the loan rate is higher, retail banks invest all their remaining deposits in loans so that $\ell_s = (1-\chi)\psi_3 m_3$. In figure 3, it is assumed that condition (7) holds. Hence, loan supply is increasing in ρ.

3.5 Miners

Similar to retail banks, there is a limited number of miners providing the fourth payment method, cryptocurrencies; see also Fernández-Villaverde and Sanches (2019). Miners act only on NM where they maximize their net consumption x_M. Their earnings are given by revenues from mining and transaction fees. If real money balances are constant, the inflation rate ψ_i is equal to the money growth rate[9]. Thus, $\delta \equiv (\psi_4 - 1)m_4$ is equal to the emission of new coins within a period. Since the value of money is one within a period, δ is equal to the revenues from mining.

Moreover, miners have costs $k(\delta)$ for issuing their own cryptocurrency[10]. The cost function $k(\delta)$ satisfies $k(0) = 0$, $k'(0) = \kappa_t > 0$, $k'(\delta) > 0$ and $k''(\delta) \geq 0$. Since mining gets more difficult by the period, the emission of a new coin gets more costly by the period[11]. The linear cost parameter κ_t captures this feature; κ_t is assumed to increase over time. By contrast, if $k'(0) = 0$, money supply would be infinite, see also Fernández-Villaverde and Sanches (2019). Thus, miners maximize their net consumption or profit, respectively, subject to their budget constraint

$$x_M = \delta + \alpha_4 \eta_4 m_4 - k(\delta). \tag{9}$$

To illustrate, let us assume $k(\delta) = \delta^3/3 + \kappa_t \delta$, with $\kappa_t = t\bar{\kappa}$. In this case, miners maximize their profit if they issue $\delta^*(t) = \sqrt{1 - t\bar{\kappa}}$. Thus, miners do not issue further coins from $t \geq 1/\bar{\kappa}$ since they would make losses. From this point, they only receive the transaction fees.

[9] See also Lagos and Wright (2005) for further details.

[10] Either all miners provide the same cryptocurrency or each miner provides another one. If all miners provide the same type, all agents demand this type. On the other hand, if every miner provides another type, all agents hold a portfolio in which every cryptocurrency has the same weight.

[11] There are several cryptocurrencies where mining gets more difficult over time, for instance, the two largest cryptocurrencies by market capitalisation, Bitcoin and Ethereum.

If $\bar{\kappa}$ is small, miners issue coins for numerous periods, see the red line in figure 4. On the other hand, if $\bar{\kappa}$ is large, miners only issue coins in the first periods, see the black line. Due to higher costs, mining is no longer profitable.

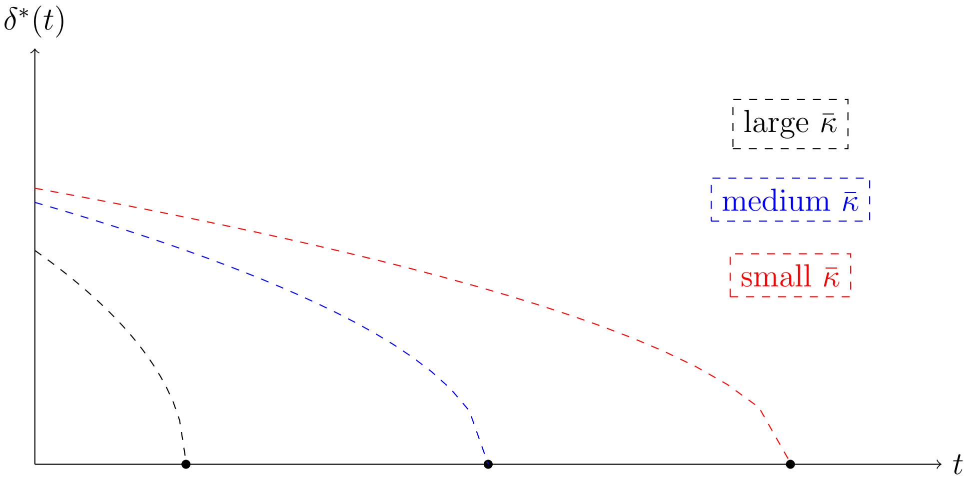

Figure 4: Issue of new coins

3.6 Central Bank

The central bank is not interested in maximizing its own net consumption x_C. Instead, the main goal is to survive by securing demand for, at least, one legal payment system, i.e. cash or CBDC. As long as cash is anonymous and has no fee, $\eta_1 = 0$, the budget constraint of the central bank is

$$\underbrace{(\psi_1 - 1)\left[m_1 + \frac{\chi\psi_3 m_3}{\psi_1}\right] + \alpha_2\eta_2 m_2}_{\text{assets}} = \underbrace{x_C + (1-\psi_2)m_2}_{\text{liabilities}}. \tag{10}$$

If $\psi_1 > 1$, assets are given by the emission of cash, the reserve requirement and the transaction fee from a CBDC. Liabilities, on the other hand, are given by net consumption and the interest rate payment for a CBDC, at least if $\psi_2 < 1$.

4 Environment with(out) a CBDC

After considering the environment in the previous chapter, we distinguish between two cases now: in case (A) the central bank has only cash available. In case (B), on the other hand, the central bank has also a CBDC available.

4.1 Environment without a CBDC

In case (A) the central bank has only cash available. It is assumed that a money contraction is impossible since issued bank notes cannot be retrieved. Otherwise, if $\psi_1 < 1$, cash is interest-bearing and a CBDC does not provide further monetary policy measures compared to cash. To secure demand for cash, the only option for the central bank is to stop the emission of further cash. In this case, the return for cash is at least zero. Nevertheless, there are storage costs since cash gets discounted by β. Thus, $\Omega_1 = \frac{1-\beta}{\beta} > 0$ holds and q_1^* is not traded. Since the Friedman rule cannot be implemented, the first best allocation is missed.

If the cost differences, Λ_3^1 and Λ_4^1, are positive for $\psi_1 > 0$ and negative for $\psi_1 = 0$, retail banks and miners have to adjust the storage cost and transaction fee of their payment systems as soon as the central bank stops the emission of further cash. Thus, they cannot maximize their profits by choosing m_3 and δ furthermore. Since trade probabilities are exogenous, there is no incentive to underbid the conditions for cash to possibly increase the market share. Thus, both groups try to match the conditions for cash so that the overall cost difference is zero, $\Lambda_3^1 = \Lambda_4^1 = 0$. In this case, cash, deposits and cryptocurrencies would be used simultaneously in equilibrium. To achieve $\Lambda_3^1 = \Lambda_4^1 = 0$, retail banks and miners have two options: either they charge no fee so that they are able to offer a lower interest rate or they offer a high interest rate so that they are able to charge a fee.

Retail banks are able to offer $1/\psi_3 = \frac{\Gamma}{1-\alpha_3\eta_3}$. In this case, they make zero profits and their cost term, $\frac{\Omega_3}{\alpha_3} + \eta_3$, is in its minimum. Implementing the maximal interest rate, $1/\psi_3 = \frac{\Gamma}{1-\alpha_3\eta_3}$, in the cost term yields $\left(\frac{1}{\alpha_3} - \eta_3\right)\left(\frac{1}{\beta\Gamma} - 1\right)$. Thus, for $\Gamma > 1/\beta$, retail banks charge no fee, $\eta_3 = 0$, since their cost term is smaller compared to the case where they charge a fee, $\eta_3 > 0$. Since the loan business is profitable, it makes sense to focus on it. For $\Gamma = 1/\beta$, retail banks are indifferent. Finally, for $\Gamma < 1/\beta$, retail banks charge a fee. Since the loan business is less profitable, it makes sense to focus on a fee for deposits.

Assuming $\Gamma = 1/\beta$ and $\eta_3 = 0$, retail banks have to offer an interest rate for deposits of

$$\Lambda_3^1 = 0 \quad \Leftrightarrow \quad \frac{1}{\psi_3} = \frac{1}{\beta + \alpha_3\left(\frac{1-\beta}{\alpha_1}\right)}.$$

Here, $\alpha_1 = \alpha_3$ implies $1/\psi_3 = 1$. If trade probabilities are equal, retail banks have to match the return for cash. On the other hand, if $\alpha_1 > \alpha_3$, retail banks have to offer a positive return to compensate private agents for the lower trade probability. Things are reversed for $\alpha_1 < \alpha_3$. Even if $\alpha_3 \to 0$, retail banks do not go bankrupt. In this case, they have to offer $1/\psi_3 = 1/\beta$, which is, due to $\Gamma = 1/\beta$ and $\eta_3 = 0$, possible. Since $1/\psi_3 = 1/\beta$, there are no storage costs so that α_3 does not affect the cost term anymore.

Things are similar for miners. Since the value for new issued coins is one within a period, miners get no interest payments on new issued coins. This is equal to $\Gamma = 1$, see eq. (5) and (9). Thus, miners always charge a fee and have to offer

$$\Lambda_4^1 = 0 \quad \Leftrightarrow \quad \frac{1}{\psi_4} = \frac{1}{\beta + \alpha_4\left(\frac{1-\beta}{\alpha_1} - \beta\eta_4\right)}.$$

As long as $\alpha_1 \geq \alpha_4$, miners have to offer a positive return on cryptocurrencies to compensate private agents for charging a fee by retracting their coins, $\delta < 0$. Miners make no losses as long as $x_M \geq 0$, which implies $\psi_4 \geq 1 - \alpha_4\eta_4$. Thus, miners are able to match the conditions for cash as long as

$$\frac{1}{\beta + \alpha_4 \left(\frac{1-\beta}{\alpha_1} - \beta\eta_4\right)} \leq \frac{1}{1 - \alpha_4\eta_4} \quad \Leftrightarrow \quad \eta_4 \geq \frac{\alpha_1 - \alpha_4}{\alpha_1\alpha_4}.$$

Since $\psi_4 = 1 - \alpha_4\eta_4 \geq \beta$, there is no space for η_4 if $\alpha_4 < \beta\alpha_1$. In this case, miners go bankrupt since they are not able to compensate private agents for the low trading probability and the transaction fee for cryptocurrencies. Thus, $\alpha_4 \geq \beta\alpha_1$ turns out to be a necessary condition for an equilibrium with cryptocurrencies. The lower bound, $\beta\alpha_1$, increases with the discount rate: if agents value the future more, the interest rate is even more important.

4.2 Environment with a CBDC

In case (B) the central bank has a CBDC available. Since a CBDC is interest-bearing, there are more monetary policy measures for the central bank to ensure demand for legal payment systems. In general, a CBDC is superior to cash if

$$\Lambda_2^1 > 0 \quad \Leftrightarrow \quad \frac{1}{\psi_2} > \frac{1}{\hat{\psi}_1} \equiv \frac{1}{\alpha_2 \left[\beta\left(\frac{1}{\alpha_2} - \eta_2\right) + (1-\beta)\left(\frac{1}{\alpha_1}\right)\right]}.$$

Even if the central bank offers a fully anonymous CBDC without a transaction fee by choosing $\eta_2 = 0$, the central bank has to ensure that the return for a CBDC exceeds a lower bound, $1/\hat{\psi}_1$. Otherwise, the central bank has no higher power compared to the case without a CBDC. As long as the return for a CBDC exceeds the lower bound, a CBDC displaces cash.

As mentioned above, as long as $\Gamma = 1/\beta$, retail banks charge no fee and are able to offer $1/\psi_3 = 1/\beta$. Thus, retail banks are always able to match the conditions for a CBDC; even if $\eta_2 = 0$. Due to $\Lambda_3^2 = 0$, a CBDC and deposits are used simultaneously in equilibrium. Now, miners have to offer

$$\Lambda_4^2 = 0 \quad \Leftrightarrow \quad \frac{1}{\psi_4} = \frac{1}{\beta + \alpha_4 \left[\frac{\psi_2 - \beta}{\alpha_2} - \beta(\eta_4 - \eta_2)\right]},$$

while they are able to offer $1/\psi_4 = 1/(1 - \alpha_4\eta_4)$. Thus, miners are able to match the conditions for a CBDC as long as

$$\frac{1}{\beta + \alpha_4 \left[\frac{\psi_2 - \beta}{\alpha_2} - \beta(\eta_4 - \eta_2)\right]} \leq \frac{1}{1 - \alpha_4\eta_4}$$
$$\Leftrightarrow \quad \frac{1}{\psi_2} \leq \frac{1}{\hat{\psi}_4} \equiv \frac{1}{\alpha_2 \left[\beta\left(\frac{1}{\alpha_2} - \eta_2\right) + (1-\beta)\left(\frac{1}{\alpha_4} - \eta_4\right)\right]}.$$

This is possible since $1/\hat{\psi}_4 > 1/\hat{\psi}_1$ holds, at least for $\eta_2 = 0$ and $\alpha_4 > \beta\alpha_1$; see also chapter 4.1. Now, for $\eta_2 = 0$, the upper bound decreases to

$$\frac{1}{\hat{\psi}_4} = \frac{1}{\beta + (1-\beta)\left(\frac{\alpha_2}{\alpha_4} - \alpha_2\eta_4\right)}.$$

Since $(1-\beta)\left(\frac{\alpha_2}{\alpha_4} - \alpha_2\eta_4\right) > 0$, there is always a space for the central bank to drive miners out of the market. Thus, if the CBDC rate is inside

$$\frac{1}{\hat{\psi}_4} < \frac{1}{\psi_2} \leq \frac{1}{\beta},$$

miners go bankrupt since they are not able to compensate private agents for the transaction fee. All in all, the central bank has a higher power with a CBDC to displace private payment systems since it can be interest-bearing. Table 1 summarizes the results for case (A) and (B).

case	(A)	(B)
instrument for the central bank	money growth rate of zero ($\Omega_1 > 0$ still holds)	increasing the CBDC rate ($\Omega_2 = 0$ is possible)
reference for retail banks and miners	conditions for cash ($\Lambda_3^1 = \Lambda_4^1 = 0$)	conditions for CBDC ($\Lambda_3^2 = \Lambda_4^2 = 0$)
payment systems used in equilibrium	if $\alpha_4 \geq \beta\alpha_1$: cash, deposits, cryptocurrencies	if $1/\psi_2 \leq 1/\hat{\psi}_4$: CBDC, deposits, cryptocurrencies
	if $\alpha_4 < \beta\alpha_1$: cash, deposits	if $1/\psi_2 > 1/\hat{\psi}_4$: CBDC, deposits

Table 1: Case (A) and (B)

Proposition 4: *(i) In case (A), cash, deposits and cryptocurrencies are used. For $\alpha_4 < \beta\alpha_1$, only cash and deposits are used. (ii) In case (B), cash is replaced by a CBDC. For $1/\psi_2 > 1/\hat{\psi}_4$, only a CBDC and deposits are used.*

5 Welfare Analysis

In the last step, we should discuss whether providing a CBDC improves welfare. Welfare is defined as the sum of consumption on DM plus net consumption on NM,

$$W = \sum_{t=0}^{\infty} \beta^t \left(\sum_{i=1}^{4} \alpha_i \Delta_i + x_P + x_B + x_E + x_M + x_C \right).$$

Implementing the budget constraints (2), (5), (8), (9) and (10) in the welfare function yields[12]

$$W = \frac{\sum_{i=1}^{4} \alpha_i \bar{\Delta}_i + f(\ell) - \ell}{1 - \beta}.$$

[12]The growth rates for cash and cryptocurrencies are zero in steady state.

Here, $\bar{\Delta}_i \equiv u(q_i) - c(q_i)$ is the trade surplus without a transaction fee. Since transaction fees are a revenue as well as a cost for a specific group they do not affect welfare directly. Nevertheless, transaction fees affect consumption on DM and thus welfare in an indirect way. Things are similar for the loan rate affecting consumption on NM. If $\psi_i = \beta$ and $\eta_i = 0$, the trade surplus is at its maximum, $\bar{\Delta}_i = \frac{\sigma}{1-\sigma}$. If the central bank, retail banks and miners were all able to offer such conditions, money holdings would be equal across all payment systems and welfare would be at its overall maximum,

$$W^* = \frac{\left(\frac{1}{J}\right)\left(\frac{\sigma}{1-\sigma}\right) + \frac{[(1-\chi)\beta]^{1-\varepsilon}}{1-\varepsilon} - (1-\chi)\beta}{1-\beta}.$$

But W^* cannot be reached since the central bank cannot implement $\psi_1 = \beta$ for cash, while miners are not able to offer $\psi_4 = \beta$ and $\eta_4 = 0$ simultaneously for cryptocurrencies. Thus, as long as $\psi_i = \beta$ and $\eta_i = 0$, the trade surplus and the amount of loans are at their maximum but cash and cryptocurrencies are not used in equilibrium so that the trade probability is below $1/J$. On the other hand, if $\psi_i > \beta$ and $\eta_i > 0$, the trade probability is maximal, but the trade surplus is less, $\bar{\Delta}_i < \frac{\sigma}{1-\sigma}$ holds. Moreover, loans are below $(1-\chi)\beta$ in this case.

In the next step, welfare between case (A) and (B) should be compared. In both cases welfare is below W^*. In case (A) only cash is available. For $\alpha_4 \geq \beta\alpha_1$, retail banks and miners are able to match the conditions for cash so that $\Lambda_3^1 = \Lambda_4^1 = 0$. Thus, welfare is

$$W_A = \frac{(\alpha_1 + \alpha_3 + \alpha_4)\bar{\Delta}_A + f(\ell_A) - \ell_A}{1-\beta},$$

where $\bar{\Delta}_A$ and ℓ_A are the trade surplus and the amount of loans in case (A). Since $\Lambda_3^1 = \Lambda_4^1 = 0$, money holdings and thus the trade surplus are equal across all three payment systems.

In case (B) a CBDC is available. If the central bank chooses $1/\psi_2 > 1/\hat{\psi}_4$, only retail banks are able to match the conditions for a CBDC while miners go bankrupt. As long as a CBDC and deposits are used simultaneously in equilibrium, welfare in case (B) is

$$W_B = \frac{(\alpha_2 + \alpha_3)\bar{\Delta}_B + f(\ell_B) - \ell_B}{1 - \beta}.$$

Since the central bank forces retail banks to increase the interest rate for deposits compared to (A), demand for deposits and hence the trade surplus in a trade with deposits increase, $\bar{\Delta}_B > \bar{\Delta}_A$. Moreover, since the amount of deposits increases, loans also do from ℓ_A to ℓ_B, see figure 5[13].

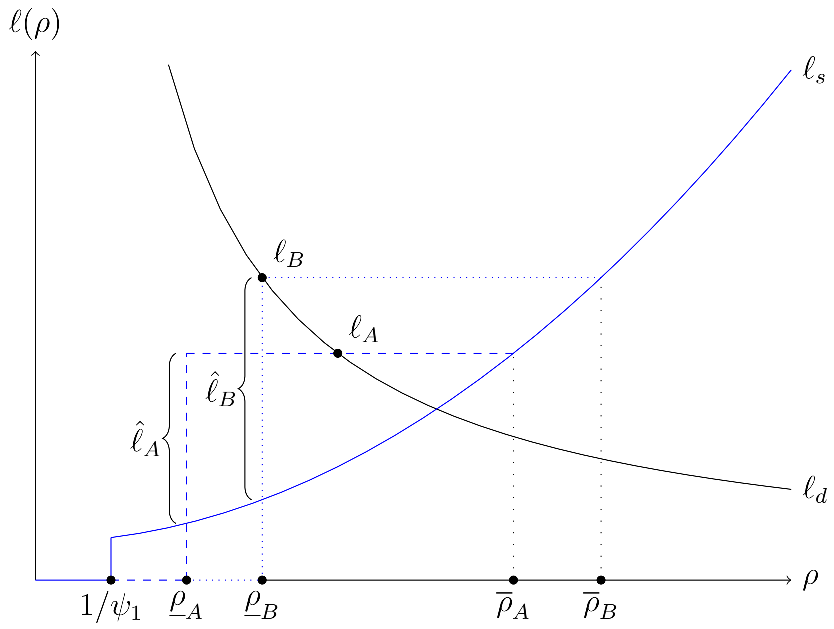

Figure 5: The loan market (with a CBDC)

[13]If retail banks face further payment systems, they have to offer a certain deposit rate to ensure $\Lambda_3^{-i} \geq 0$ and need a minimum loan rate, $\underline{\rho}$, to offer the required deposit rate. Due to the higher deposit rate, deposits and loans increase for $\rho = \underline{\rho}$, see $\hat{\ell}$. For $\rho > \underline{\rho}$, retail banks make profits again. Since the amount of loans is still above the optimal amount of loans without further payment systems, retail banks do not increase loan supply. Finally, for $\rho > \overline{\rho}$, loan supply increases since it is profit maximizing again (dashed and dotted line merge with the blue line). In addition, the amount of loans is maximal for $\underline{\rho} = \underline{\rho}_B$. For $\underline{\rho} > \underline{\rho}_B$, the amount of loans decreases again.

On the one hand consumption in a trade with deposits and gains from investment capital increase, on the other the number of trades on DM decreases since only a CBDC and deposits are used in equilibrium, at least if $\alpha_2 < \alpha_1 + \alpha_4$. The gain exceeds the loss if $W_B > W_A$ which requires

$$\underbrace{\alpha_2 \bar{\Delta}_B}_{\substack{\text{trades with} \\ \text{a CBDC}}} + \underbrace{\alpha_3(\bar{\Delta}_B - \bar{\Delta}_A)}_{\substack{\text{add. trade surplus} \\ \text{for deposits}}} + \underbrace{[f(\ell_B) - \ell_B] - [f(\ell_A) - \ell_A]}_{\substack{\text{add. surplus from} \\ \text{investment capital}}} > \underbrace{(\alpha_1 + \alpha_4)\bar{\Delta}_A}_{\substack{\text{missed trades} \\ \text{with cash and} \\ \text{cryptocurrencies}}}. \tag{11}$$

As long as the trade probabilities for cash and cryptocurrencies are limited, condition (11) is fulfilled for sure. In this case, the loss due to the missed trades with cash and cryptocurrencies is manageable since only a few trades disappear. If the trade probabilities are large, on the other hand, there is the risk that the loss exceeds the gain from using a CBDC. Thus, the sign of the welfare effect mainly depends on the circulation of the different payment systems. This result confirms the findings from Fuchs and Michaelis (2022) who emphasize that the welfare effect mainly depends on the fraction of agents using digital money.

Proposition 5: *For $1/\psi_2 > 1/\hat{\psi}_1$, consumption in a trade with a CBDC and deposits as well as gains from investment capital increases. On the other hand, for $\alpha_2 < \alpha_1 + \alpha_4$, the number of trades on DM decreases since trades with cash and cryptocurrencies disappear. Thus, welfare only increases if the gain exceeds the loss, see condition* (11).

6 Conclusion

This work provides some useful insights about the competition between legal and private payment systems. First, the trade probability (transaction fee) is of importance for money demand if the storage costs are high (low). Second, even if every payment system has a positive expected net value, private agents only use the payment system(s) with the highest expected net value. Third, loan supply only increases in the loan rate if money demand is in an elastic area. Fourth, since mining gets more difficult over time, miners only issue coins up to a certain point in time. And fifth, and most important, the central bank is able to secure demand for legal payment systems by providing an interest-bearing CBDC. In this way, retail banks and miners are forced to match the conditions for a CBDC to avoid runs. Retail banks will do this. By contrast, miners go bankrupt. Alongside that, welfare increases if the gain due to higher consumption and loans exceeds the loss due to fewer trades.

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